On the Coefficient of Variation and its Inverse

MSC2010 Classification : 62D05, 62E17,62E20,62N02,62N03

ABSTRACT

In this paper we are studying the coefficient of variation of a continuous random variable and some other concepts, like its inverse (symbol ICv or Cv^{-1}) or its square inverse (symbol $ICv^2 = q$), e.tc. Basically, we try to develop the asymptotic sampling distribution of the inverse coefficient of variation $ICV = CV^{-1}$. This distribution is used to infer statistically significant results for the coefficient of variation or the inverse coefficient of variation of a random variable X without making a hypothesis for the population distribution of the variable X. We are focused on same cases of random variables following specific distributions, dealing with the related parameters of those distributions. E.g. Our attention is mostly focused on extracting results (confidence intervals, hypothesis testing), for parameters of Gamma distribution, Weibull distribution e.tc.

Some examples are given in order to illustrate the particulars of the behavior of the ICv and ICv^2, related with the above-mentioned concepts: Confidence intervals, testing hypothesis, etc. for the random variable X and so on.

RESUME

Dans cet article, nous étudions le coefficient de variation d'une variable aléatoire continue et de quelques autres concepts, tels que son inverse (symbole ICv ou Cv^{-1}) ou son inverse carré (symbole $ICv^{-2} = q$), etc. Fondamentalement, nous essayons de développer la distribution d'échantillonnage asymptotique du coefficient de variation inverse $ICV = CV^{-1}$. Cette distribution est utilisée pour déduire des résultats statistiquement significatifs pour le coefficient de variation, ou le coefficient de variation inverse d'une variable aléatoire X sans émettre d'hypothèse pour la distribution de la population de la variable X. Nous concentrons sur les mêmes cas de variables aléatoires suivant des distributions spécifiques. Traitant des paramètres connexes de ces distributions. Par exemple, notre attention se concentre principalement sur l'extraction des résultats (intervalles de confiance, tests d'hypothèses) pour les paramètres de distribution Gamma, distribution de Weibull, e.tc.

Quelques exemples sont donnés pour illustrer les particularités du comportement de ICv et ICv^2 , liées aux concepts susmentionnés : intervalles de confiance, hypothèse de test, etc.

Keywords: Inverse coefficient of variation, sampling, estimation, testing hypothesis

1. **Introduction.**

The coefficient of variation (CV) is the standard deviation ratio to the mean value. CV has many applications that are used in statistical analysis. A fundamental property is that it helps analyzing the distribution of random variables, through sampling, with a very simple and easy to handle procedure. Distributions of random variables that are symmetrical, increasing and decreasing, as well as combinations of them have been studied, Farmakis (2003). The knowledge of the distribution of the CV and its inverse is calculated by sampling data, Farmakis (2003, 2010), Sharma & Krishna (1994). The distribution of the sampling CV is quite complicated and requires complex procedures to be determined. Thus, we often resort to its asymptotic form through a big sample, Sharma & Krishna (1994).

2. **Asymptotic distribution of ICV.**
3. **The case of Gamma distribution.**
4. **The case of Weibull distribution.**
5. **Inference for the shape parameter of Gamma, b.**
   1. **Point Estimation.**
   2. **Interval Estimation.**
6. **Exponentiality test.**
   1. **Gamma distribution.**
   2. **Weibull distribution.**
7. **Examples with data.**
8. **Conclusions**

For Gamma and Weibull distribution, reliability is a function of a, b. For known α, it is a 1-1 function of b and therefore a 1-1 function of ICv (b). Therefore, ICv's tests and confidence intervals can be used to develop tests and intervals for reliability.
For lognormal distribution, where the standard deviation of $logx$ is b, conclusions can be drawn.
$ICv = (\exp(b^2) - 1)^{-\frac{1}{2}}$ that is independent of θ. So, when θ is unknown, tests and confidence intervals can easily be extracted for the sample ICv.

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