Time-varying end-of-month effects in German currency in circulation

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1 Introduction

The increasing availability of long economic time series poses a challenge for seasonal and calendar adjustment in practice as seasonal and calendar movements now may exhibit changes which are barely identifiable over shorter time periods. The popular X-11 and ARIMA model-based seasonal adjustment methods are capable of dealing with a fair amount of moving seasonality. The respective pretreatment regression models, however, are based on the assumption of constant calendar effects and, thus, do not allow for a direct estimation of "moving" calendar effects. As a compromise, official statistics usually follow a rather pragmatic approach based on dividing the entire observation span into several (potentially overlapping) sub-spans and performing separate seasonal and calendar adjustments on each of those, in line with Item 6.2 of the recommendations [1]. Still, the key question remains: can calendar effects be generally assumed to stay constant over at least several years?

Applying structural time series (STS) models, which are considered acceptable in Item 3.1 [1], this paper exemplarily studies monthly currency in circulation for Germany as of January 1980 up to February 2018. More precisely, we consider the index of notional stocks [2], that is outstanding amounts adjusted for changes not induced by transactions such as reclassifications, exchange rate variations and other revaluations, which is set to 100 for December 2008. Since data is reported at the last banking day of a month, it is likely to be affected by the particular weekday which the last day of a given month falls onto. We refer to this effect as the end-of-month (EOM) effect.

The remainder of this paper is organised as follows. Section 2 describes the particular STS model for currency in circulation, including details on the construction of EOM regression variables. Section 3 reports the estimation results with special emphasis on differences between constant and time-varying EOM regression coefficients. Finally, Section 4 draws some conclusions.

2 Methods

Let $\{y_t\}$ be a time series with τ observations per year. Adopting standard notations [3], we consider the univariate STS model

$$y_t = \mu_t + \gamma_t + \mathbf{w}_t^\top \boldsymbol{\theta} + \mathbf{x}_t^\top \boldsymbol{\delta}_t + \varepsilon_t, \tag{1}$$

where $\{\mu_t\}$ is a local linear trend given by

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, & \eta_t &\overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\eta^2\right), \quad \sigma_\eta^2 > 0, \\ \beta_t &= \beta_{t-1} + \zeta_t, & \zeta_t &\overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\zeta^2\right), \quad \sigma_\zeta^2 > 0, \end{aligned}$$

and $\{\gamma_t\}$ is a trigonometric seasonal component defined as $\gamma_t = \sum_{j=1}^{\lfloor \tau/2 \rfloor} \gamma_{j,t}$ with

$$\begin{pmatrix} \gamma_{j,t} \\ \gamma_{j,t}^\star \end{pmatrix} = \begin{pmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{pmatrix} \begin{pmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^\star \end{pmatrix} + \begin{pmatrix} \omega_{j,t} \\ \omega_{j,t}^\star \end{pmatrix},$$

where $\{\omega_{j,t}\}$ and $\{\omega^{\star}_{j,t}\}$ are two mutually uncorrelated Gaussian white noise processes with zero means and common variance $\sigma^2_\omega \geq 0$ at each seasonal frequency, that is

$$\begin{pmatrix} \omega_{j,t} \\ \omega^{\star}_{j,t} \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\omega & 0 \\ 0 & \sigma^2_\omega \end{pmatrix} \right].$$

Furthermore, $\mathbf{w}_t$ and $\mathbf{x}_t$ are vectors of intervention and EOM regression variables, respectively, at time t. The intervention effects $\boldsymbol{\theta}$ are assumed to be constant over time, whereas the EOM effects are assumed to follow a multivariate random walk according to

$$\boldsymbol{\delta}_t = \boldsymbol{\delta}_{t-1} + \boldsymbol{\upsilon}_t, \quad \boldsymbol{\upsilon}_t \overset{iid}{\sim} \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\right). \tag{2}$$

Thus, the EOM effects are time-constant as well if $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} = \mathbf{0}$ and time-varying in any other case. Finally, $\{\varepsilon_t\}$ is Gaussian white noise with zero mean and finite variance $\sigma^2_\varepsilon \geq 0$. After being put into state space form, Eq. (1) is estimated by the Kalman filter and smoother with a diffuse initialisation.
To construct $\mathbf{x}_t$, let $\mathcal{D} = \{\text{MON}, \text{TUE}, \text{WED}, \text{THU}, \text{FRI}, \text{SAT}, \text{SUN}\}$ be the set of the seven weekdays and $D_t \in \mathcal{D}$ an indicator of the weekday which the last day of month t falls onto.[1] Then, we define six EOM contrasts by

$$D_{i,t} = \begin{cases} 1, & D_t = i \\ -1, & D_t = \text{SUN} \\ 0, & \text{otherwise} \end{cases}, \quad i \in \mathcal{D} \setminus \{\text{SUN}\},$$

calculate centred versions $\tilde{D}_{i,t} = D_{i,t} - \bar{D}_i$, where $\bar{D}_i$ is the month-specific average of $D_{i,t}$,[2] and finally set $\mathbf{x}_t = (\tilde{D}_{\text{MON},t} \cdots \tilde{D}_{\text{SAT},t})^\top$. Accordingly, the vector of EOM effects becomes $\boldsymbol{\delta}_t = (\delta_{\text{MON},t} \cdots \delta_{\text{SAT},t})^\top$ and we assume for convenience that the EOM effects follow independent random walks, that is $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}$ is diagonal in Eq. (2).

3 Results

Using the STAMP 8.3 module [5] of OxMetrics 7, we fit Eq. (1) to logged currency in circulation and refer to the STS models with time-constant and time-varying EOM effects as the CON-EOM and VAR-EOM models, respectively. To identify interventions, we first run the automatic outlier detection routine in default mode for the CON-EOM model; afterwards, we iteratively use standard t-tests to check for significant level and white noise auxiliary residuals and manually add the proper intervention variable once an absolute t-value exceeds 4. The interventions identified this way are then specified in both STS models.
Tab. 1 reports the estimated intervention effects, which barely differ between the two STS models, and the estimated EOM effects for the CON-EOM model. The latter turn out to be significantly positive for Friday and Saturday, significantly negative for Tuesday and Wednesday and not significantly different from zero for Monday and Thursday, resulting in a derived negative EOM effect for Sunday. An intuitive explanation may be that private consumers tend to increase cash reserves at the end of the week due to planned weekend shopping and other activities. Tab. 2 reports the

[1] In the case of public holidays at the end of a month the following rule is applied: if the last day of month t falls onto a (1) Monday which is a public holiday, then $D_t = \text{SUN}$; (2) Tuesday and the preceding Monday is a public holiday, then $D_t = \text{MON}$; (3) Thursday and the following Friday is a public holiday, then $D_t = \text{FRI}$.

[2] To obtain the centred EOM contrasts, we use data as of January 1969 up to December 2020 and the JDemetra+ plug-in TransReg [4].

Table 1: Estimated end-of-month (EOM), additive outlier (AO) and level break (LB) effects from Eq. (1) fitted to German currency in circulation (italicised interventions have been identified automatically).

	CON-EOM model			VAR-EOM model		
Effect	Estimate	t-value	p-value	Estimate	t-value	p-value
EOM MON	−0.0008	−1.4366	0.1516	N/A	N/A	N/A
EOM TUE	−0.0031	−5.4899	< 0.0001	N/A	N/A	N/A
EOM WED	−0.0026	−4.6052	< 0.0001	N/A	N/A	N/A
EOM THU	−0.0008	−1.3435	0.1789	N/A	N/A	N/A
EOM FRI	0.0037	6.8277	< 0.0001	N/A	N/A	N/A
EOM SAT	0.0019	3.5337	0.0005	N/A	N/A	N/A
EOM SUN	−0.0017	derived		N/A	N/A	N/A
AO Mar 1983	0.0220	4.2508	< 0.0001	0.0166	4.1745	< 0.0001
LB Dec 1988	0.0311	4.3909	< 0.0001	0.0271	4.7190	< 0.0001
LB Jul 1990	0.0571	8.0633	< 0.0001	0.0609	10.5652	< 0.0001
AO Dec 1992	0.0303	5.8869	< 0.0001	0.0268	6.8317	< 0.0001
AO Sep 2001	−0.0358	−4.9265	< 0.0001	−0.0331	−5.3498	< 0.0001
AO Oct 2001	−0.0848	−8.4250	< 0.0001	−0.0871	−9.4227	< 0.0001
AO Nov 2001	−0.1806	−14.0945	< 0.0001	−0.1764	−14.6250	< 0.0001
LB Dec 2001	−0.5112	−32.3559	< 0.0001	−0.5129	−34.4431	< 0.0001
AO Jan 2002	−0.0488	−9.3867	< 0.0001	−0.0469	−12.7172	< 0.0001
AO Dec 2003	0.0258	4.9948	< 0.0001	0.0204	5.1623	< 0.0001
LB Oct 2008	0.0444	6.1997	< 0.0001	0.0493	8.4810	< 0.0001
LB Jan 2009	−0.0840	−11.6692	< 0.0001	−0.0783	−13.4952	< 0.0001
LB Jan 2014	−0.0564	−7.8945	< 0.0001	−0.0494	−8.4590	< 0.0001

estimated variances of the components which are common to both STS models. As for the intervention effects, the estimates are very similar except for the white noise component which vanishes in the VAR-EOM model. This most likely compensates for adding the random walk EOM effects, the estimated disturbance covariance matrix of which is given by:

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{v}} = 10^{-8} \times \mathrm{diag}\begin{pmatrix}1.5899 & 3.6446 & 6.1947 & 0.6931 & 6.2896 & 2.2330\end{pmatrix}.$$

Fig. 1 shows the estimated EOM effects for the CON-EOM and VAR-EOM models. At the beginning, the two types of EOM effects have the same sign but the time-varying ones are approximately twice as strong on average. From 1990 onwards, they decrease more or less smoothly and eventually converge to zero for each weekday but Monday, for which the estimated EOM effect changes sign during 2005 and increases steadily to almost 0.06% until 2018.
Given the intuitive explanation of the estimated constant EOM effects, these findings may indicate that private consumers have changed their payment behaviour gradually over the last decades as they nowadays tend to ask for less cash at the end of the week, providing implicitly empirical evidence of an increasing popularity of cashless payment systems.

Table 2: Estimated level, slope and seasonal disturbance as well as white noise variances from Eq. (1) fitted to German currency in circulation (in units of 10^{-5}).

Model	$\hat{\sigma}^2_\eta$	$\hat{\sigma}^2_\zeta$	$\hat{\sigma}^2_\omega$	$\hat{\sigma}^2_\varepsilon$
CON-EOM	1.1120	0.6748	0.0038	0.9170
VAR-EOM	1.5030	0.6825	0.0031	0.0000

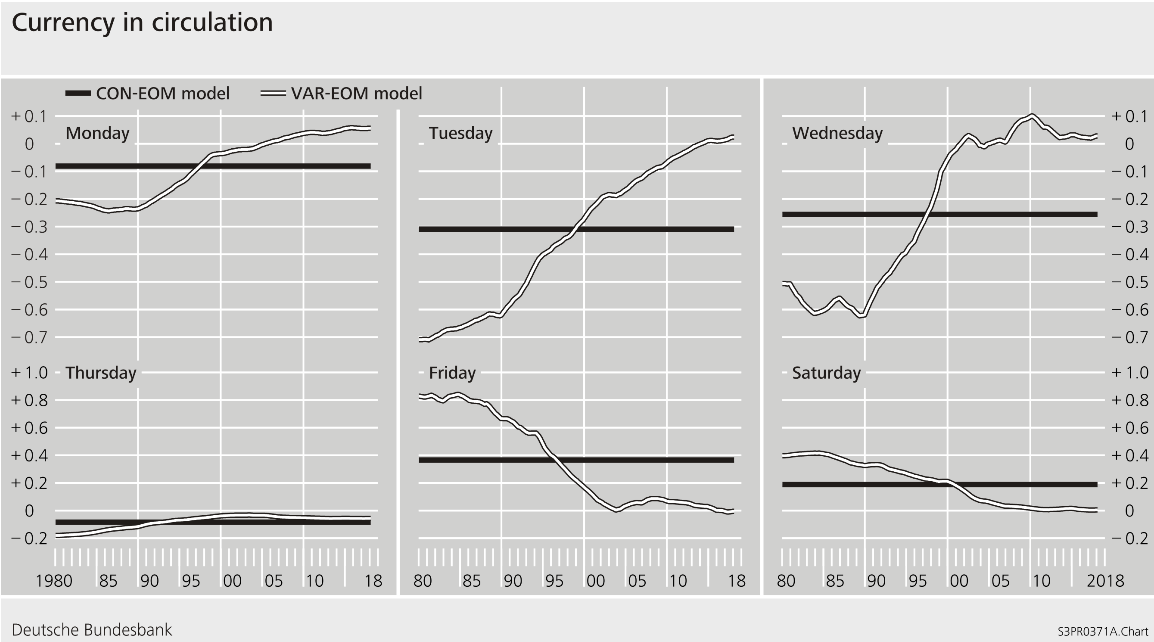

Figure 1: Estimated end-of-month (EOM) effects from Eq. (1) fitted to German currency in circulation (as a percentage).

4 Conclusions

When considering the entire observation span, the assumption of constant EOM effects in monthly German currency in circulation seems to be invalid. Instead, the EOM effects exhibit both smooth transitions (e.g. for Thursday) and sudden changes (e.g. for Wednesday and Friday). Thus, even the pragmatic "sub-span approach" [1] appears inappropriate for this particular time series. In general, further research is needed on how to deal with calendar effects which evolve rather rapidly over time.

References

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