DEUTSCHE BUNDESBANK
EUROSYSTEM

Time-varying end-of-month effects in German currency in circulation

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Research question

Payment instruments in Germany

- BBk consumer surveys (2008, 2011, 2014, 2017) ⇝ Micro data
- Cashless vs cash ⇝ Gradually increasing vs decreasing popularity
- Key drivers ⇝ Age, income, level of education, place of payment, type of transaction

Verification from macro data?

- Currency in circulation ⇝ Notional stocks
- Monthly reporting ⇝ Last banking day
- Potential influence ⇝ Weekday of last day of a month

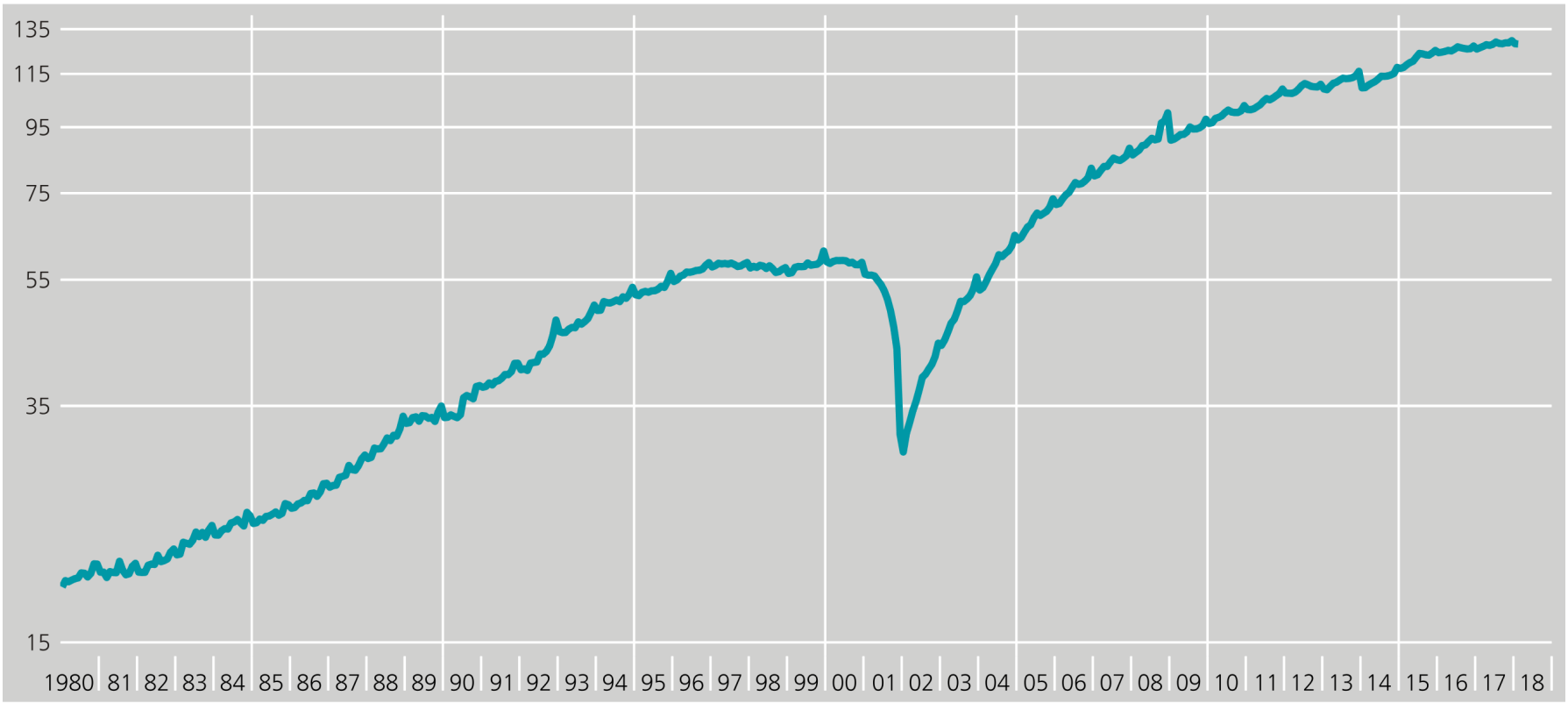
Key question

- End-of-month (EOM) effects ⇝ Decreasing absolute sizes?

Data

Currency in circulation

December 2008 = 100, monthly, log scale



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Data model & trend

$$
\begin{aligned}
\log \text{CIC}_t &= \mu_t + \gamma_t + \mathbf{w}_t^\top \boldsymbol{\theta} \\
&+ \mathbf{x}_t^\top \boldsymbol{\delta}_t + \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \sigma_\varepsilon^2 \geqslant 0
\end{aligned}
$$

- $\{\mathbf{w}_t\} \rightsquigarrow$ Interventions
- $\{\mathbf{x}_t\} \rightsquigarrow$ EOM regression variables

Local linear trend

$$
\begin{aligned}
\mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\eta^2\right), \quad \sigma_\eta^2 > 0 \\
\beta_t &= \beta_{t-1} + \zeta_t, \qquad\qquad \zeta_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\zeta^2\right), \quad \sigma_\zeta^2 > 0
\end{aligned}
$$

Stochastic trigonometric seasonal

$$\begin{aligned} \gamma_t &= \sum_{j=1}^{[\tau/2]} \gamma_{j,t} \\ \begin{pmatrix} \gamma_{j,t} \\ \gamma^{\star}_{j,t} \end{pmatrix} &= \begin{pmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{pmatrix} \begin{pmatrix} \gamma_{j,t-1} \\ \gamma^{\star}_{j,t-1} \end{pmatrix} + \begin{pmatrix} \omega_{j,t} \\ \omega^{\star}_{j,t} \end{pmatrix} \end{aligned}$$

Distributional assumption

$$\begin{pmatrix} \omega_{j,t} \\ \omega^{\star}_{j,t} \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\mathbf{0}, \sigma^2_{\omega} \times \mathbf{I}\right), \quad \sigma^2_{\omega} > 0$$

EOM regression variables

EOM contrasts

$$D_{i,t} = \begin{cases} 1, & D_t = i \\ -1, & D_t = \text{SUN}\,, \\ 0, & \text{otherwise} \end{cases} \quad i \in \{\text{MON}, \text{TUE}, \ldots, \text{SAT}\}$$

– $\{D_t\} \rightsquigarrow$ Weekday of last day of a month

STS model ingredient

$$\mathbf{x}_t = \begin{pmatrix} \tilde{D}_{\text{MON},t} & \tilde{D}_{\text{TUE},t} & \cdots & \tilde{D}_{\text{SAT},t} \end{pmatrix}^\top$$

– $\{\tilde{D}_{i,t}\} \rightsquigarrow \{D_{i,t}\}$ centred around month-specific means

EOM effects & model variants

EOM effects

$$\boldsymbol{\delta}_t = \boldsymbol{\delta}_{t-1} + \boldsymbol{\upsilon}_t, \quad \boldsymbol{\upsilon}_t \overset{iid}{\sim} \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}^{(\text{diag})}\right)$$

Competitors

- $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}^{(\text{diag})} = \mathbf{0} \rightsquigarrow$ CON-EOM model
- $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}^{(\text{diag})} \neq \mathbf{0} \rightsquigarrow$ VAR-EOM model

Interventions (I/II)
Identification

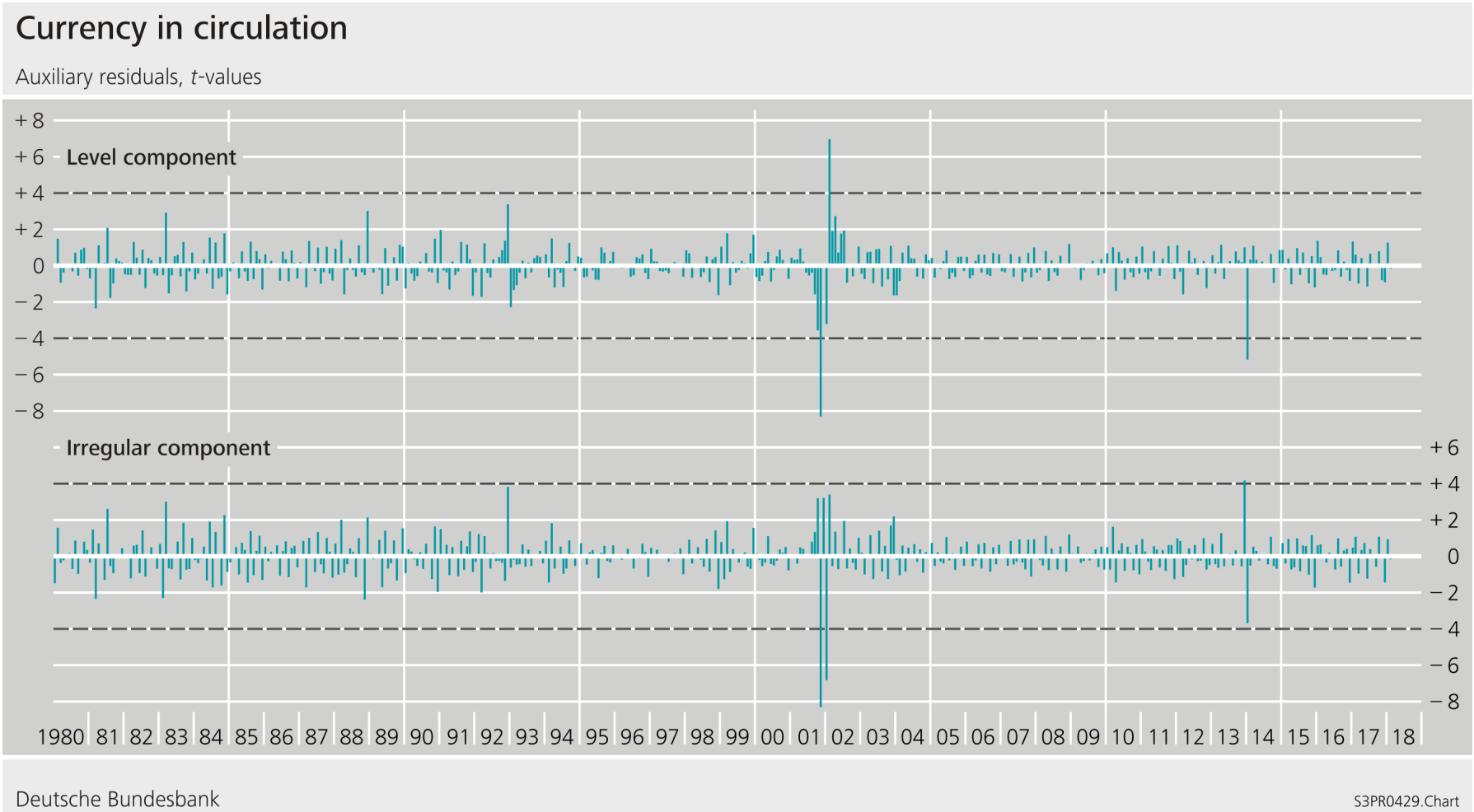
CON-EOM model

- Automatic outlier detection $\rightsquigarrow$ Default mode
- Iterative user specification $\rightsquigarrow$ Level & irregular auxiliary residuals ($|t$-value$| > 4$)

VAR-EOM model

- User specification $\rightsquigarrow$ Identified CON-EOM model outliers

Interventions (II/II)
Illustration: CON-EOM model after automatic outlier detection



Estimated intervention effects

Estimated additive outlier (AO) and level break (LB) effects*

	CON-EOM model			VAR-EOM model		
	Estimate	t-value	p-value	Estimate	t-value	p-value
AO 03.1983	0.022	4.251	< 0.001	0.017	4.175	< 0.001
LB 12.1988	0.031	4.391	< 0.001	0.027	4.719	< 0.001
LB 07.1990	0.057	8.063	< 0.001	0.061	10.565	< 0.001
AO 12.1992	0.030	5.887	< 0.001	0.027	6.832	< 0.001
AO 09.2001	−0.036	−4.927	< 0.001	−0.033	−5.350	< 0.001
AO 10.2001	−0.085	−8.425	< 0.001	−0.087	−9.423	< 0.001
AO 11.2001	−0.181	−14.095	< 0.001	−0.176	−14.625	< 0.001
LB 12.2001	−0.511	−32.356	< 0.001	−0.513	−34.443	< 0.001
AO 01.2002	−0.049	−9.387	< 0.001	−0.047	−12.717	< 0.001
AO 12.2003	0.026	4.995	< 0.001	0.020	5.162	< 0.001
LB 10.2008	0.044	6.120	< 0.001	0.049	8.481	< 0.001
LB 01.2009	−0.084	−11.669	< 0.001	−0.078	−13.495	< 0.001
LB 01.2014	−0.056	−7.895	< 0.001	−0.049	−8.459	< 0.001

* Underlined interventions have been identified via automatic outlier detection.

Estimated disturbance variances

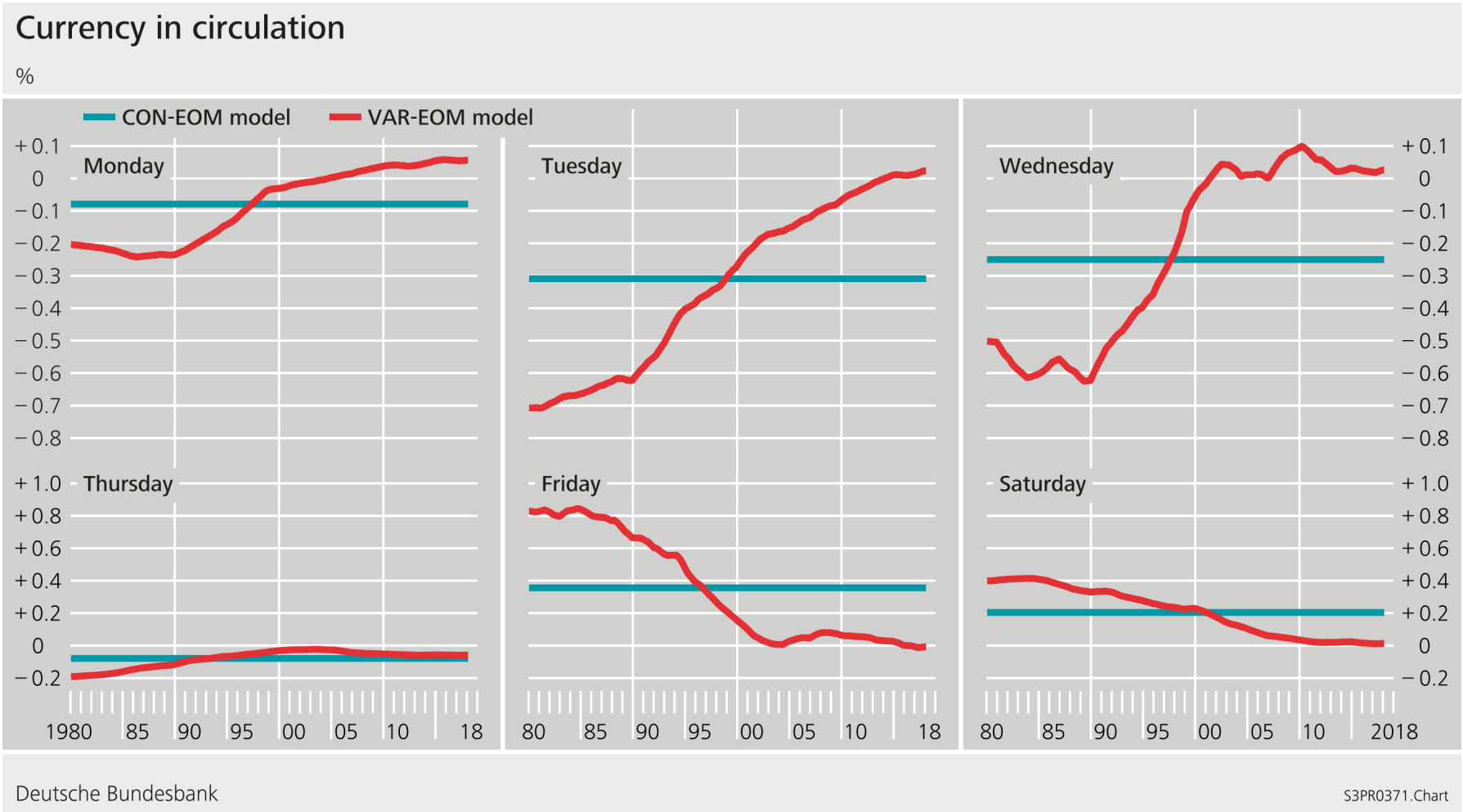
Estimated UC disturbance variances
In units of 10^{-5}

Model	$\hat{\sigma}^2_\eta$	$\hat{\sigma}^2_\zeta$	$\hat{\sigma}^2_\omega$	$\hat{\sigma}^2_\varepsilon$
CON-EOM	1.112	0.675	0.004	0.917
VAR-EOM	1.503	0.683	0.003	0.000

EOM effects (VAR-EOM model)

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{v}}^{(\text{diag})} = 10^{-8} \times \text{diag}\begin{pmatrix} 1.590 & 3.645 & 6.195 & 0.693 & 6.290 & 2.233 \end{pmatrix}$$

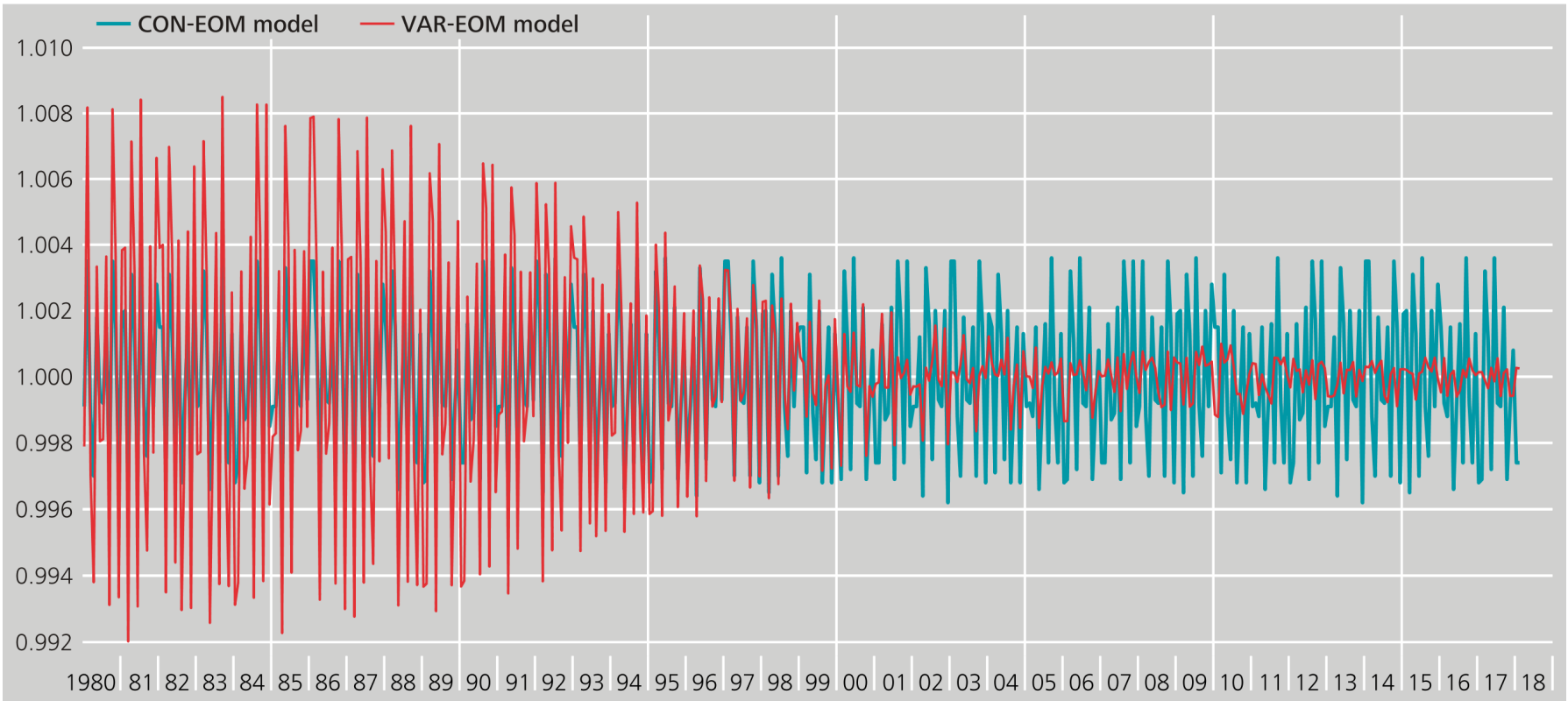
Estimated EOM effects



Estimated calendar components

Currency in circulation

December 2008 = 100, monthly, log scale



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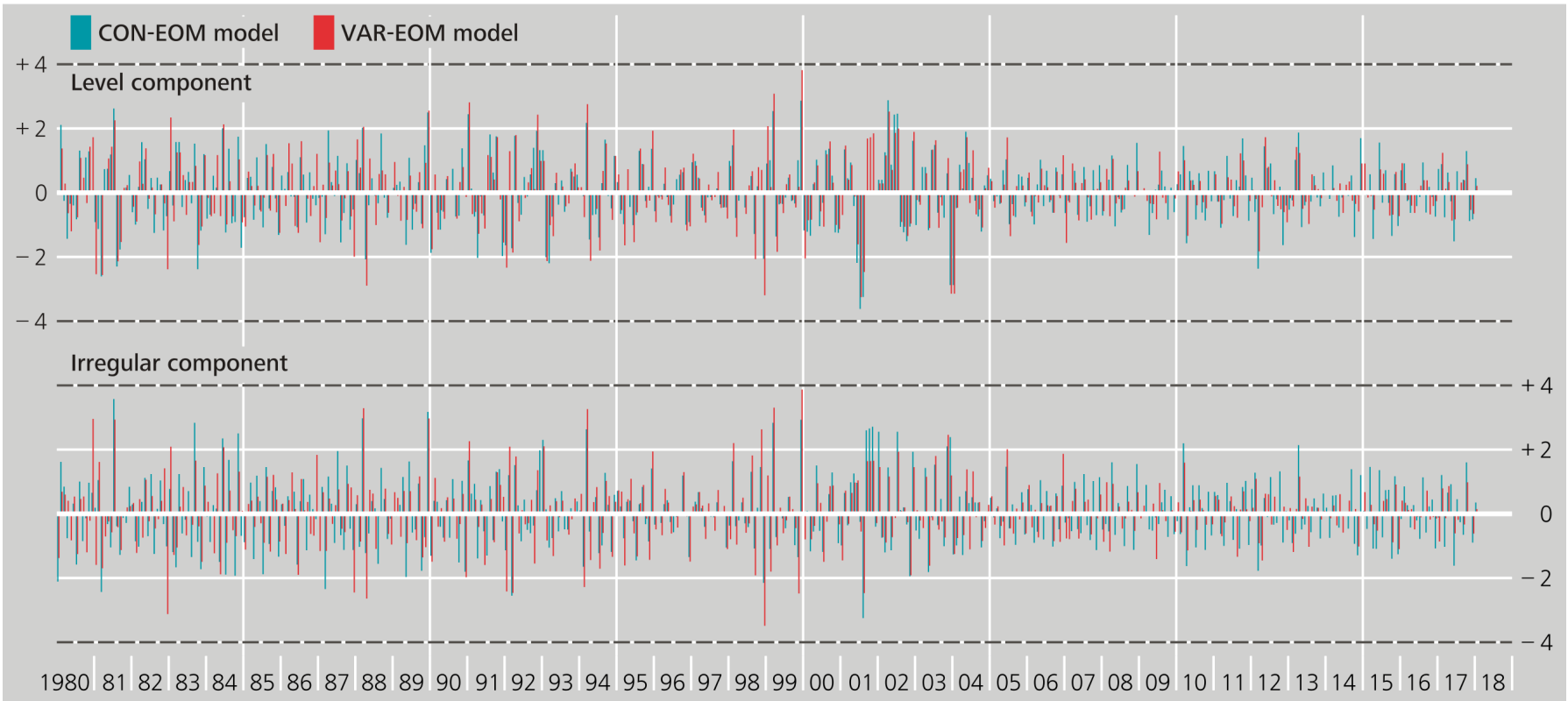
Model adequacy

Currency in circulation

Auxiliary residuals, *t*-values



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Summary

EOM effects

- Dynamics ⇝ Time-varying (rather than time-constant)
- Absolute sizes ⇝ Decrease to zero (except for MON)
- Findings ⇝ Consistent with former BBk surveys

Future research

- EOM regression variables ⇝ Other than weekday contrasts
- EOM effect dynamics ⇝ Other than random walks
- Software ⇝ Other than OxMetrics (maybe JD+?)

References (I/II)
Basic theory

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- A C Harvey (1989), Forecasting, Structural Time Series Models, and the Kalman Filter, Cambridge University Press.
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