ABSTRACT

In this paper we are studying the coefficient of variation of a continuous random variable and some other concepts, like its inverse (symbol ICv or CV−1) or its square inverse (symbol ICv2 = q), etc. Basically, we try to develop the asymptotic sampling distribution of the inverse coefficient of variation ICv = CV−1. This distribution is used to infer statistically significant results for the coefficient of variation or the inverse coefficient of variation of a random variable X without making a hypothesis for the population distribution of the variable X. We are focused on some cases of random variables following specific distributions dealing with the related parameters of those distributions. E.g. Our attention is mostly focused on extracting results (confidence intervals, hypothesis testing) for parameters of Gamma distribution, Weibull distribution etc.

Some examples are given in order to illustrate the particulars of the behavior of the ICv and ICv2, related with the above-mentioned concepts: Confidence intervals, testing hypothesis, etc. for the random variable X and so on.

Key words: Inverse coefficient of variance, Sampling, Estimation, Hypothesis Testing

1. Introduction

The coefficient of variation (CV) is the standard deviation ratio to the mean value. CV is used in studies that are used in statistical analysis. A fundamental property is that helps analyze the distribution of random variables, through sampling, with a very easy and easy to handle procedure. Distributions of random variables that are symmetrical, increasing and decreasing, as well as combinations of them have been studied, Farmakis (2003). The knowledge of the distribution of the CV and its inverse is calculated by sampling data, Farmakis (2003, 2010), Sharma & Krishna (1994). The distribution of the sampling CV is quite complicated and requires complex procedures to be determined. Thus, we often resort to its asymptotic form through a big sample, Sharma & Krishna (1994). Some symbols related to the distribution of CV and ICV are the following:

Symbols

μ,σ: mean, standard deviation for the population
ICV = σ/μ for the population inverse CV.
N : sample size
x̄, s: sample mean, sample standard deviation
c = 2−2 is the sample ICV
→ convergence in law
→ convergence in Probability
θ, β: parameters (scale, shape )
ICV(β): inverse coefficient of variation versus parameter b
\( ^{\wedge} \) suggests that the parameter crowned by this mark is an estimator
p(x): power function
f(x): probability density function of X
F(x): distribution function of the square X

2. Asymptotic distribution of ICV

We adopt the index \( \tilde{c} \) as a sample estimator of ICV, a symbol.
CV sampling value properties:
• The method of the moments gives that \( \tilde{c} \) is an asymptotically unbiased and consistent estimator of ICV
• \( \sqrt{N} (c − ICV) \) has asymptotically standardized normal distribution.
The second property is useful for testing hypotheses for CV or ICV.

Example:

Apply to Survival / Renewal Theory (survival theory)
The widely used model is exponential because of the simplicity in its mathematical expression. Its major drawback, is that its constant failure rate, is rarely accurate.

For Gamma and Weibull:

\[
\begin{align*}
\text{Gamma: } & ICV = \sqrt{\tilde{b}}, \quad Cdf(x; \theta, b) = \text{gamma}(\frac{x}{\tilde{b}}; b) \\
\text{Weibull: } & ICV = \frac{G(1)}{G(2) - G(1)^2}, \quad Cdf(x; \theta, b) = \text{weibull}(\frac{x}{\tilde{b}}; b), \\
& G(m) = \Gamma(1 + \frac{m}{b})
\end{align*}
\]

3. The case of Gamma and Weibull

Testing for b when \( \theta \) is unknown requires the use of the statistic \( W \). Although the distribution of \( W \) does not contain \( \theta \), it is quite complicated. In many reports, the distribution is discussed, and procedures are provided for the estimation of probabilities.

Another method for hypothesis testing of b contains \( \tilde{b} \). An empirical approach to the distribution of \( b \) is in the form:

\[ g(\frac{b}{b_0}) = c\sqrt{\frac{\tilde{b}}{2}}, \] where \( g \) and \( h \) are selected so that mean and variance of both agree.

5. Conclusions on b case

To avoid solving complicated equations of maximum likelihood, a large sample can be used for testing \( H_0: b = b_0 \) (with unknown \( \theta \)) for Gamma distribution and Weibull distribution.

5.1 Point estimate

The function of inverse coefficient of variance is a 1-1 and ascending function of \( b \). Therefore, testing \( H_0: b = b_0 \) is equivalent to the \( H_0: ICV = ICV(b_0) \) versus the alternative assumptions. Using the asymptotic sample distribution of inverse CV, which does not contain the variable \( \theta \), we can develop test over ICV using the Neyman-Pearson approach.

For control \( H_0: b = b_0 \) or \( H_1: ICV(b) = ICV(b_0) \) ICV(\( \beta \)) with rejection area: \( c \geq ICV + gua(f^{-1}(a)) \) with power function:

\[ P(IVC) = gua(f) \] \[ z = \sqrt{N} \frac{ICV_0 - ICV}{ICV_0 - ICV(b_0)} \] The test is stable because \( \tilde{z} \rightarrow 1 \) when \( N \rightarrow \infty \)

Testing the equality of the parameters \( b \) (shape parameters) of the Gamma distribution or the Weibull distribution when the parameters \( \theta \) (scale parameters) are unknown can be developed in a similar way.

5.2 Interval estimate

The 2-sided symmetrical \( (1 - a) \) confidence interval for ICv or \( b \) can be obtained as follows:

\[ Pr(c - gua(1 - a)) \leq ICV \leq c + gua(1 - a)) = 1 - a \] \[ gua^{-1}(a) = \frac{1}{\tilde{b}} \] as \( N \) increases so the width of the space decreases.

6. Exponentiality control

A special case of control is usually considered exponential \( (b = 1) \) versus FFR \( (b > 1) \) in Gamma and Weibull distributions.

7. Example with data

<table>
<thead>
<tr>
<th>Gamma</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.84587</td>
<td>62880</td>
</tr>
<tr>
<td>37610</td>
<td>64610</td>
</tr>
<tr>
<td>75000</td>
<td>1.56600</td>
</tr>
<tr>
<td>3.05300</td>
<td>1.71720</td>
</tr>
<tr>
<td>1.35450</td>
<td>1.93100</td>
</tr>
<tr>
<td>1.88020</td>
<td>1.55090</td>
</tr>
<tr>
<td>1.57000</td>
<td>1.61730</td>
</tr>
<tr>
<td>1.77080</td>
<td>1.31620</td>
</tr>
<tr>
<td>1.35920</td>
<td>1.77050</td>
</tr>
<tr>
<td>3.046600</td>
<td>1.88890</td>
</tr>
<tr>
<td>1.79610</td>
<td>1.88890</td>
</tr>
<tr>
<td>1.53190</td>
<td>4.15050</td>
</tr>
<tr>
<td>59030</td>
<td></td>
</tr>
</tbody>
</table>

A random sample of 25 observations was created by Weibull distribution with \( b = 2, \theta = 4 \).

8. Highlights

• For Gamma distribution as well as for Weibull, reliability is a function of \( a, b \). For known \( a \), it is a 1-1 function of \( b \) and therefore a 1-1 function of \( ICV(b) \). Therefore, ICV’s tests and confidence intervals can be used to develop tests and intervals for reliability.

• For lognormal distribution, where the standard deviation of logx is \( b \), conclusions can be drawn. ICV = (exp(b2) − 1)(\( z^2 \)) that is independent of \( \theta \). So when \( \theta \) is unknown, tests and confidence intervals can easily be extracted for the sample ICV.

BIBLIOGRAPHY

II. Φαρμάκος Ν. (1915) «Δεικτολογία & Εφαρμογές» Ελληνική Ακαδημαϊκή Ελεκτρονική Σχολή του Χαίρετη, Κέρκυρας.
III. Φαρμάκος Ν. (1916) «Επιστημονική Απεικόνιση» Αριό Κυριακάκη Εκδόσεις Α.Ε. Θεσσαλική.