



Adaptive Matrix Interpolation for On-the-fly Reduced-Order Modeling of Turbulent Combustion

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On-the-fly reduced-order modeling reformulates the species transport equation as a matrix differential equation (MDE), which is then solved on a low-rank manifold. This approach yields a set of time-dependent subspaces: the column subspace, spanning the spatial domain, and the row subspace, capturing species composition. The evolution equations for these subspaces are derived using the dynamical low-rank approximation, wherein the full-order model is projected onto the tangent space of the low-rank manifold. However, this formulation is intrusive and does not reduce floating point operation (FLOP) counts, limiting its practical efficiency. We propose a novel formulation for integrating the species transport equation on a low-rank matrix manifold using CUR matrix interpolation. This method constructs low-rank approximations by strategically sampling representative columns and rows of the species matrix. We demonstrate that this approach enables significant computational speedups—ranging from $O(10)$ - $O(100)$ —and paves the way for developing new, efficient explicit and implicit time integration schemes for turbulent combustion. Several test cases of varying complexity are presented to highlight the effectiveness and versatility of the proposed method.

References

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