



# Differential privacy and noisy confidentiality concepts for European population statistics

NTTS 2021

Session 'Input and output privacy in official statistics', 11 March 2021

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European Commission – Eurostat

Unit F2 – Population and migration

# Outline

1. Intro: 21<sup>st</sup> century statistical confidentiality
2. Noisy concepts: bottom-up and top-down
3. Risks: averaging and exploiting constraints
4. Utility: (noise) tail wagging the (statistic) dog
5. Outro: the 2021 EU census picture

# Intro: 21<sup>st</sup> century statistical confidentiality

## 20<sup>th</sup> century lore:

- must protect **individuals**

SEX \ POB*	Total	Country	Outside
Total	42	35	7
Male	22	17	5
Female	20	18	2

\* Place of birth (POB)

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- must protect **individuals**
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- must protect **individuals**
- therefore treat **small counts...**
- ... and ensure **consistency...**
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SEX \ POB*	Total	Country	Outside
Total	42	35	7
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➔ looks easy, but is generally **neither simple nor efficient**

# Intro: 21<sup>st</sup> century statistical confidentiality

## 21<sup>th</sup> century state of the art:

- database reconstruction theorem ([Dinur and Nissim, 2003](#))

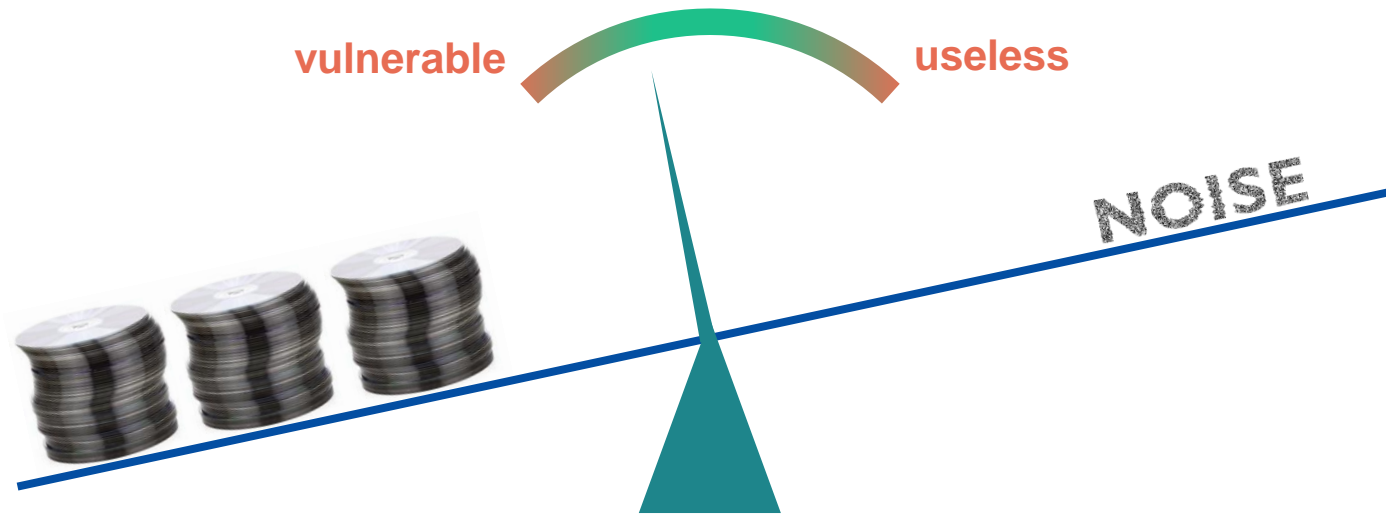
*Too many statistics, published too accurately, allow full & accurate reconstruction of all the input microdata...*

(example e.g. in [U.S. Census Bureau, 2018a, 2018b](#))

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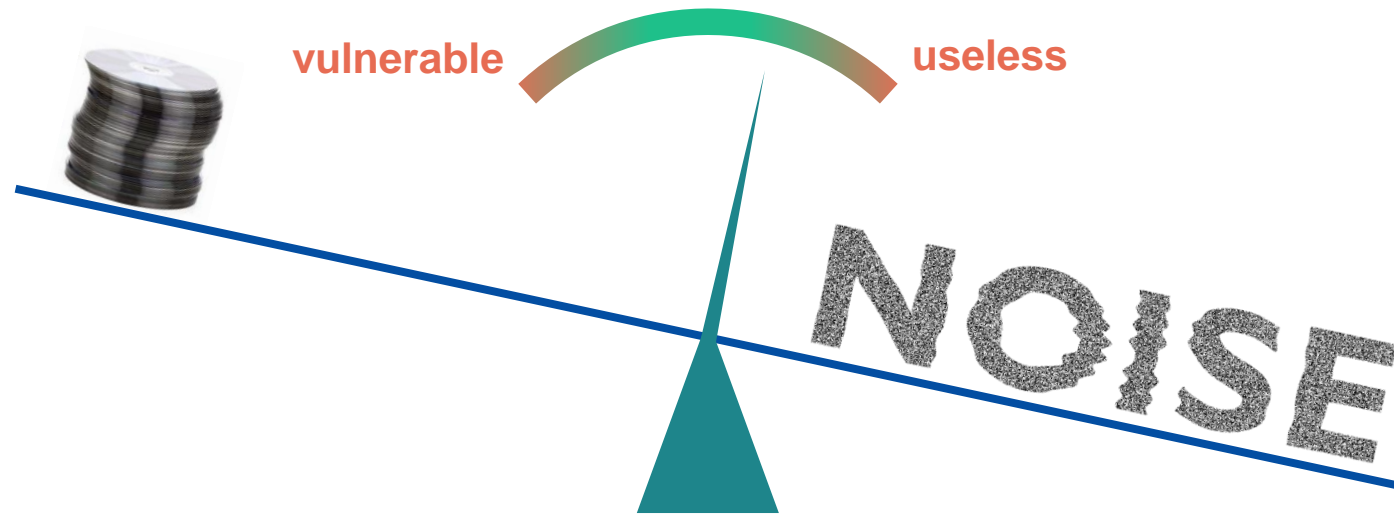
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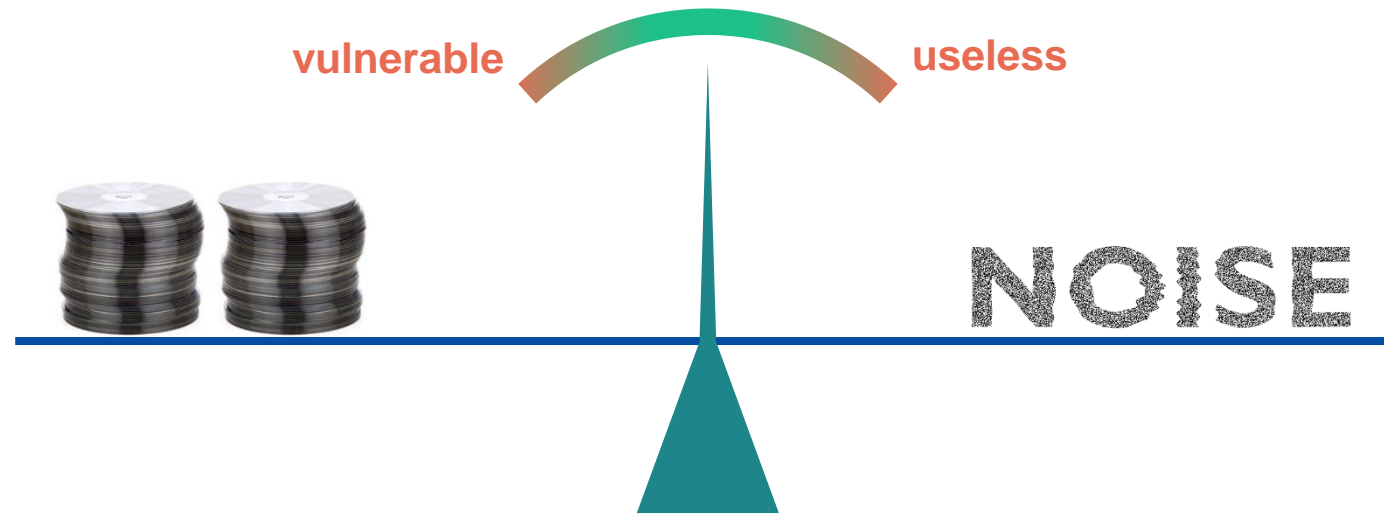




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# Noisy concepts: bottom-up

Noise in action:

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# Noisy concepts: bottom-up

Noise in action: **Is this better?**

SEX \ POB	Total	Country	Outside
Total	42	37	7
Male	23	15	4
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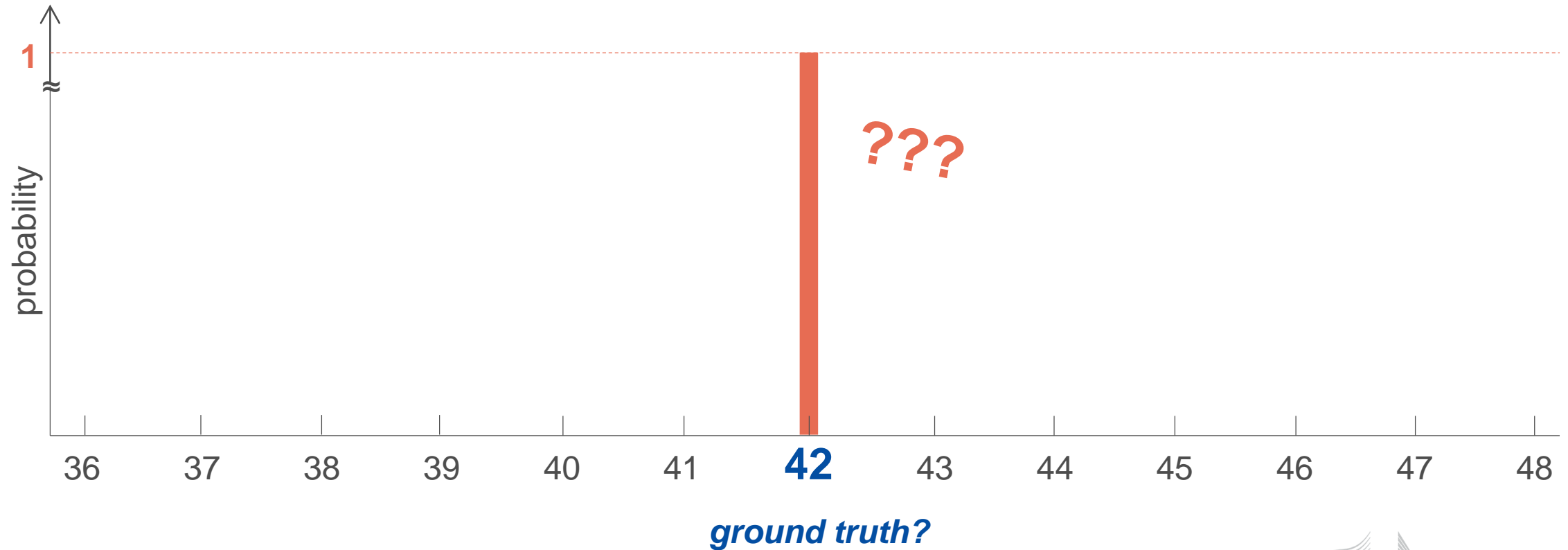
# Noisy concepts: bottom-up

... a closer look at **single statistic** level – e.g. total population in the area:

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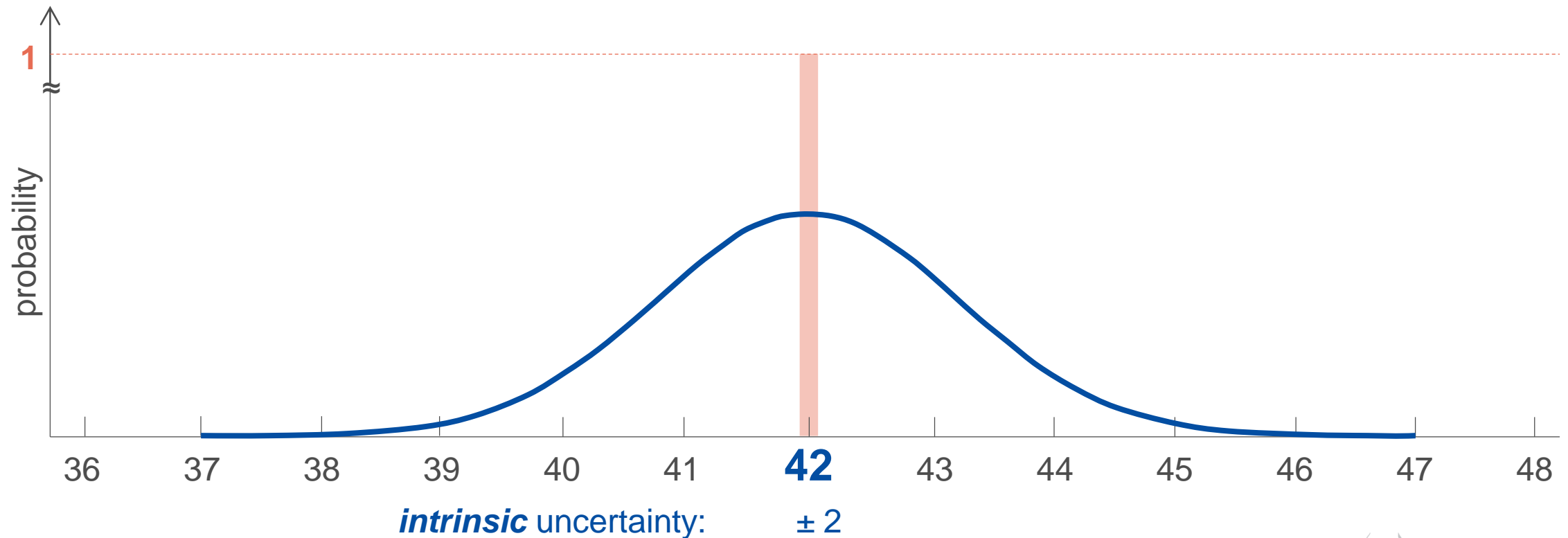
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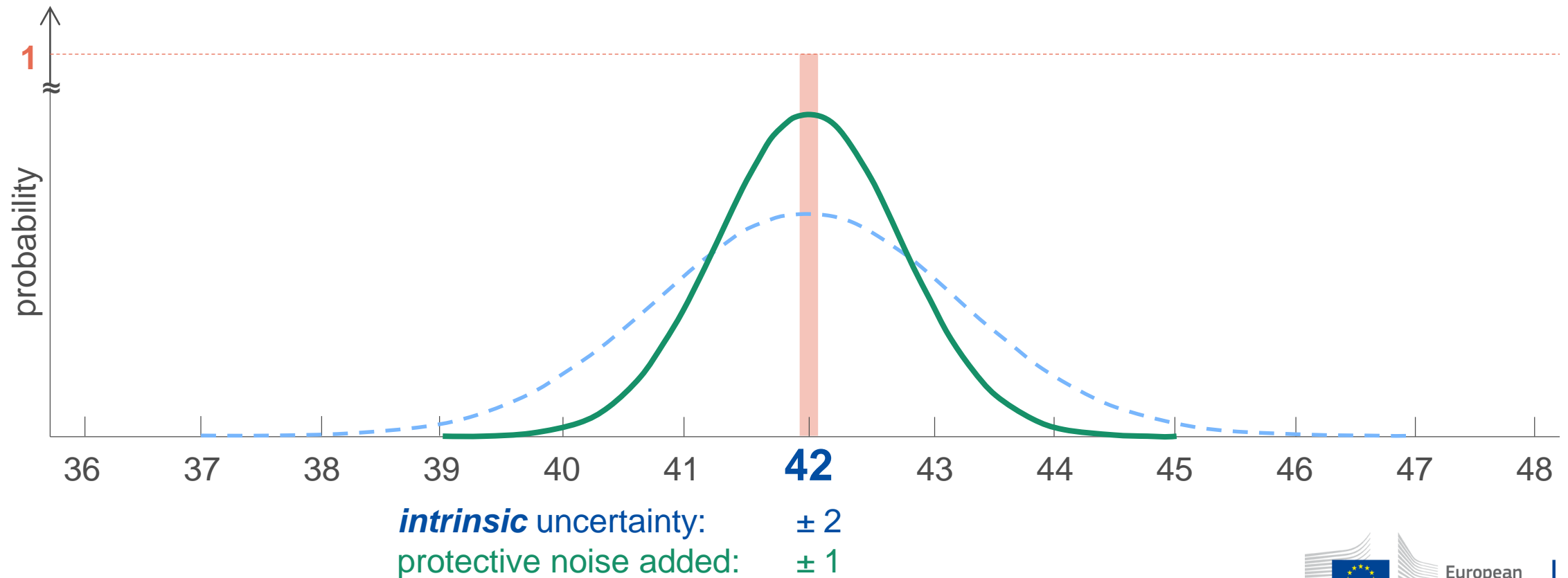
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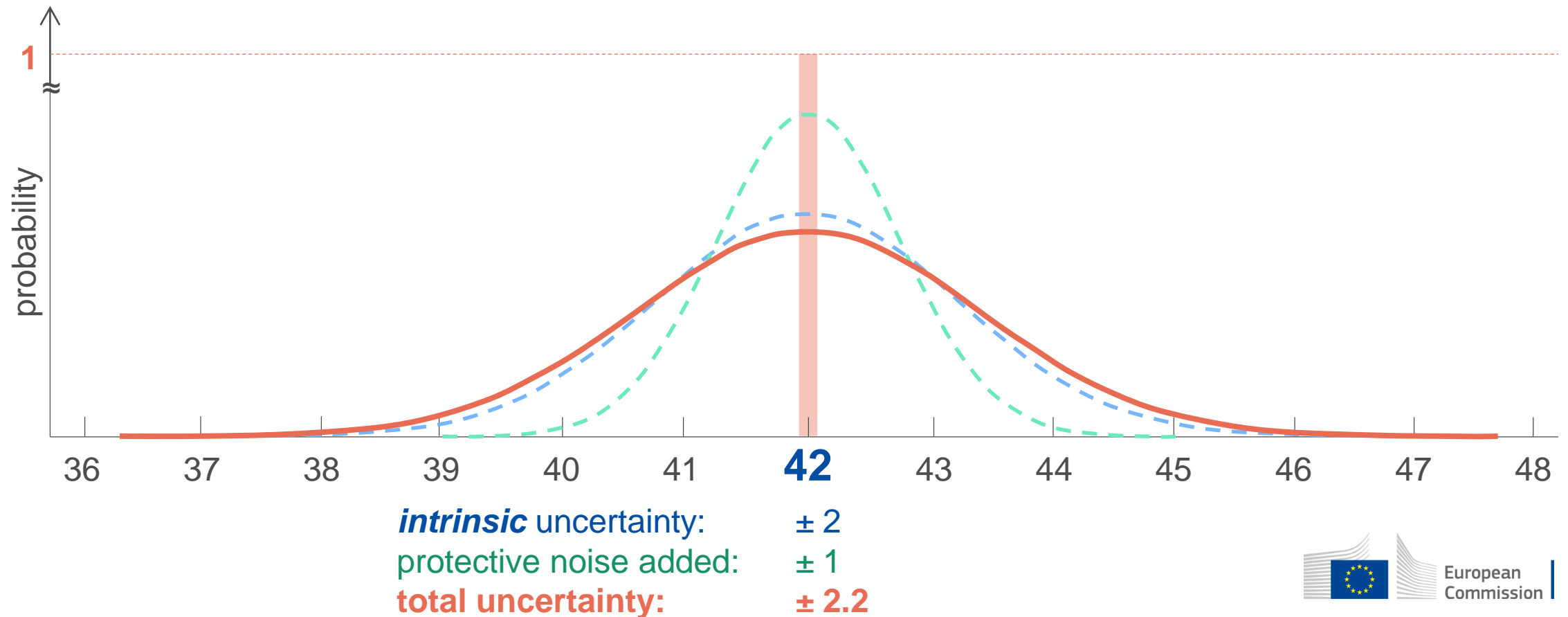
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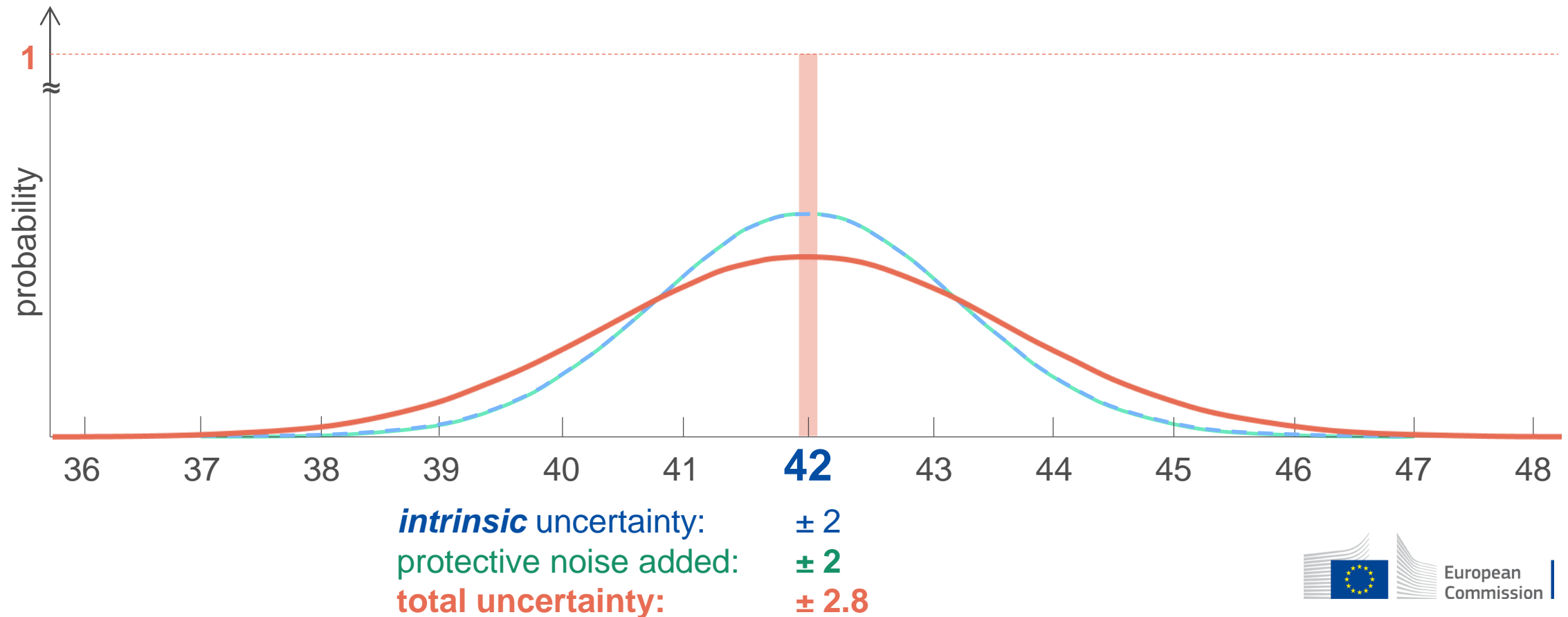
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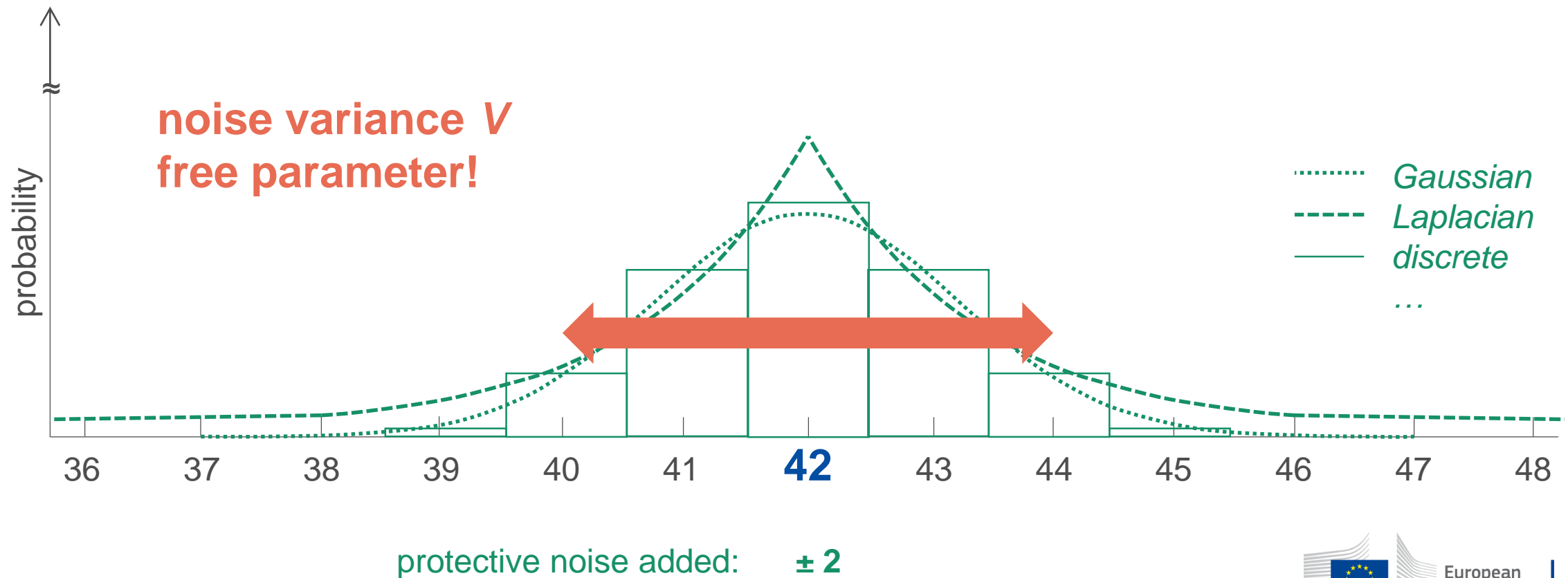
# Noisy concepts: bottom-up

... a closer look at **single statistic** level – e.g. total population in the area:



# Noisy concepts: bottom-up or *utility-driven*

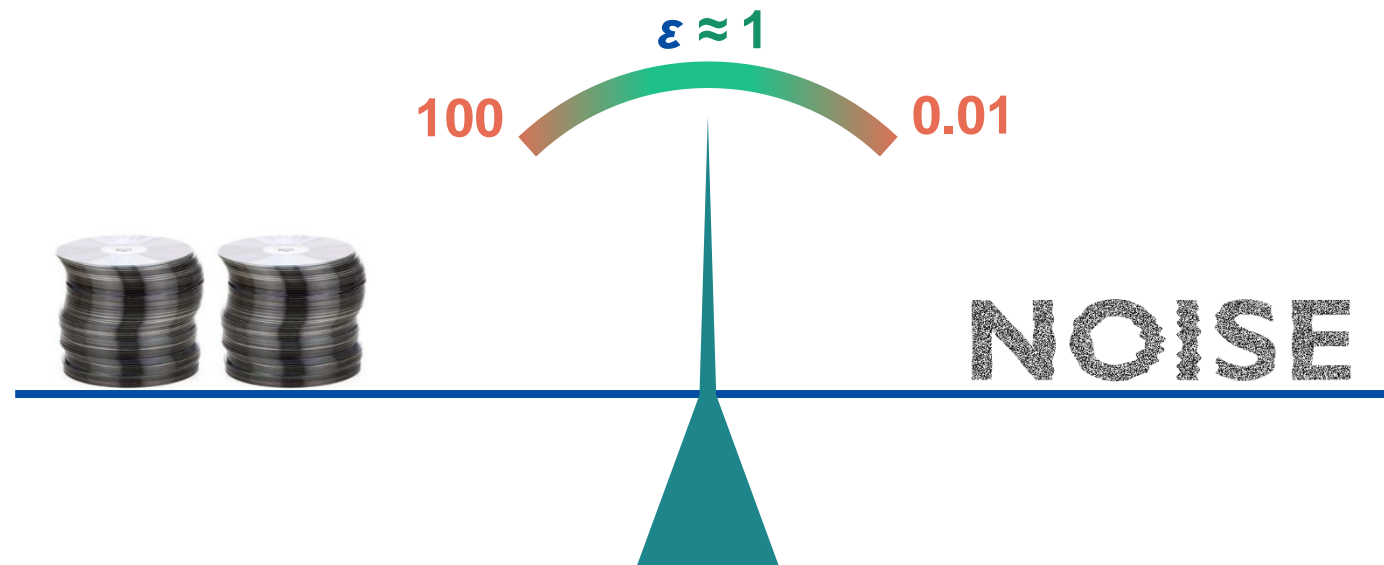
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# Noisy concepts: top-down

## Differential privacy (DP) picture:

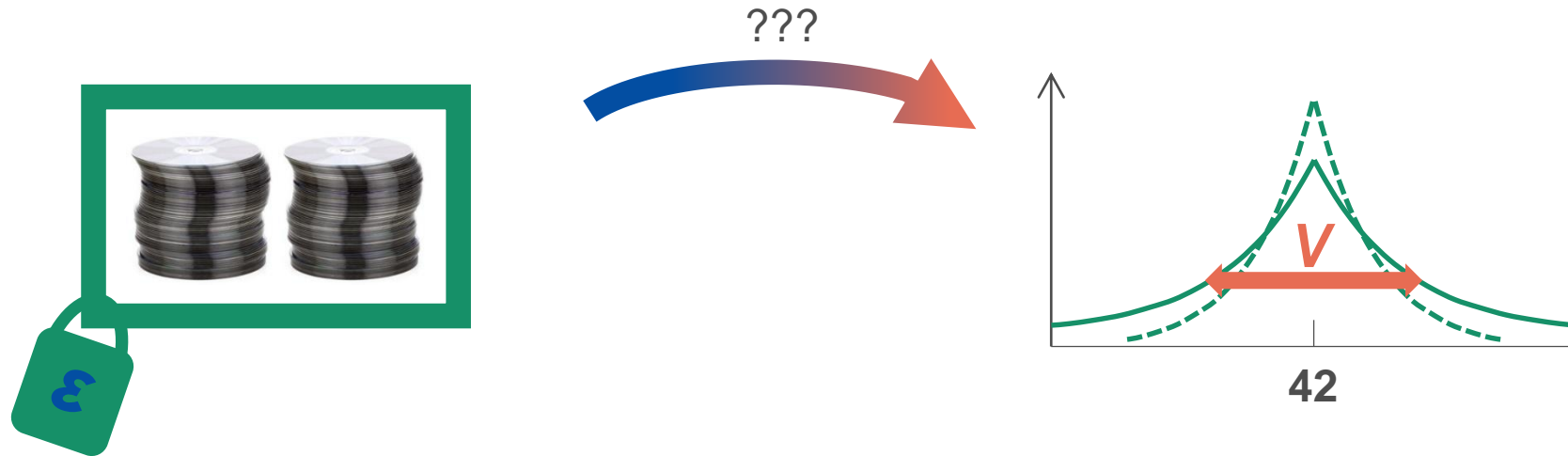
- introducing global privacy budget  $\epsilon$  ([Dwork et al., 2006](#))



# Noisy concepts: top-down or *risk-driven*

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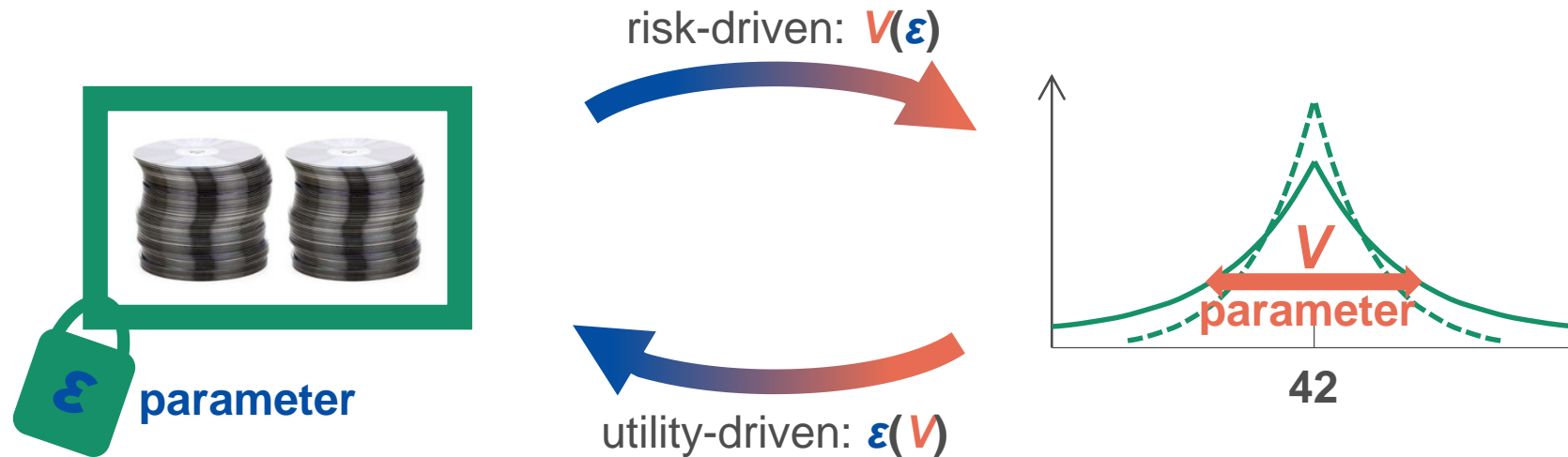
- introducing global privacy budget  $\epsilon$  ([Dwork et al., 2006](#))
- promise: strong **global privacy guarantee** ... but **local noise size**?



# Noisy concepts: top-down or *risk-driven*

## Differential privacy (DP) picture:

- introducing global privacy budget  $\epsilon$  ([Dwork et al., 2006](#))
- promise: strong **global privacy guarantee** ... but **local noise size**?



# Risks: massive averaging

- How many independent observations  $t$  of “total population” are in this table?

☐  $t = 1$

☐  $t = 2$

☐  $t = 3$

☐  $t = 4$

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- average variance:

$$\bar{V} = \frac{k}{t^2} V = \frac{9}{4^2} 1 = 0.5$$

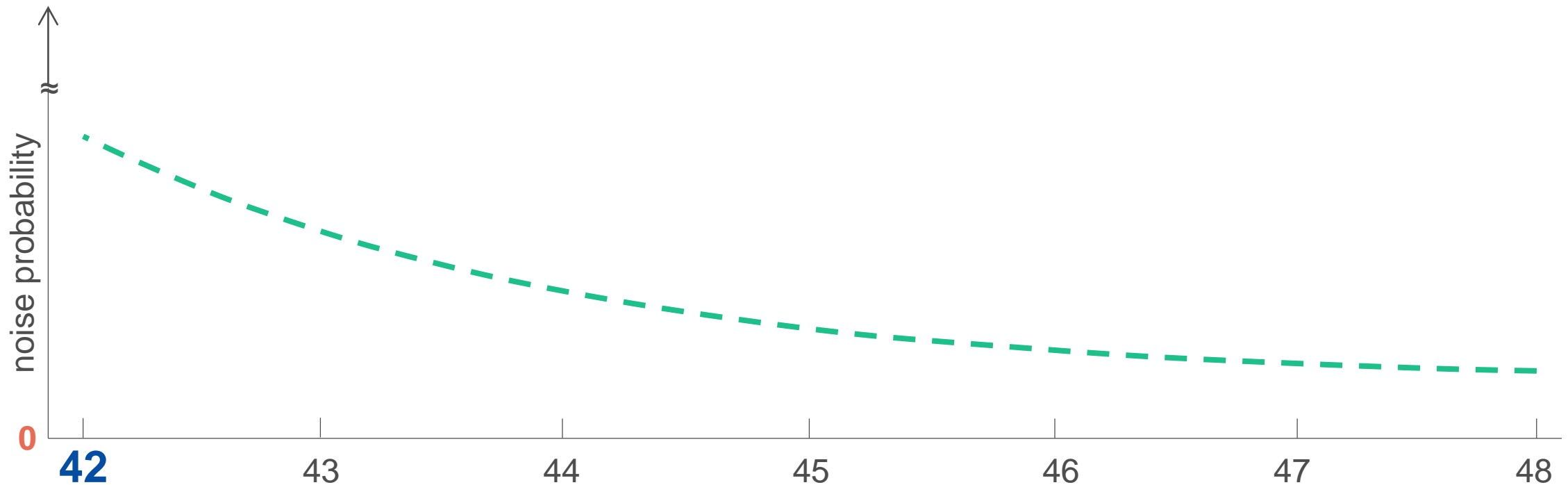
fixed by output tables

noise parameter



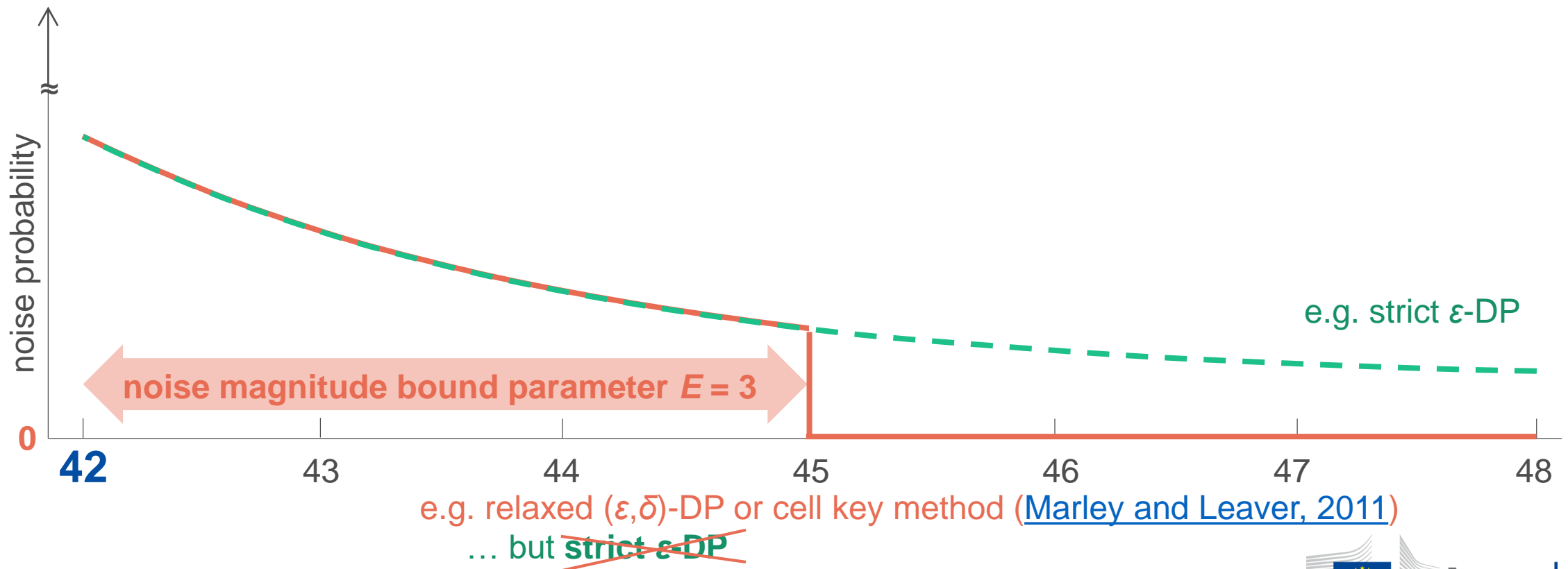
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- Noise distributions – part 2: how long is the **tail**?



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# Risks: exploiting table constraints

- Now would you bet all your money on a guess for the **true count** of the ...

- ☐ ... total population?
- ☐ ... country-born males?
- ☐ ... total females?
- ☐ ... total foreign-born?

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each count with noise variance  $V = 1$   
and noise bound  $E = 2$

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each count with noise variance  $V = 1$   
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- How often does this happen?

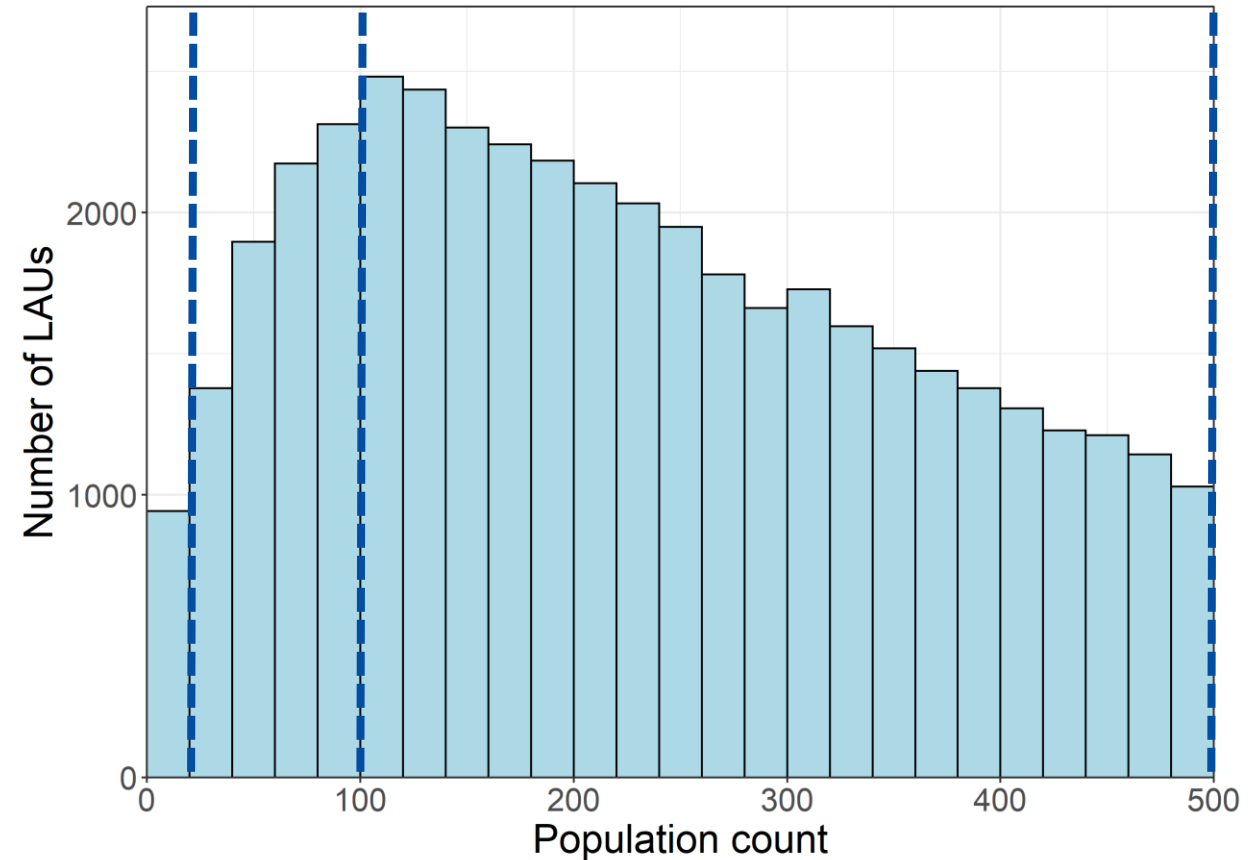
$$\text{\# of constraint } n\text{-tuples in output} \times \Pr(\text{noise} = \pm E)^n$$

fixed by output tables

fixed by noise parameters  $V$  and  $E$

# Utility: (noise) tail wagging the (statistic) dog

- 2021 EU census: ca. 110 000 **L**ocal **A**dministrative **U**nits (~ municipalities), of which
  - 43 395 with <500 people
  - 8 502 with <100 people
  - 866 with <20 people
- Could we accept here e.g.  $\Pr(|\text{noise}| > 100) = 0.1\%$  or more?
  - ☐ Yes
  - ☐ No

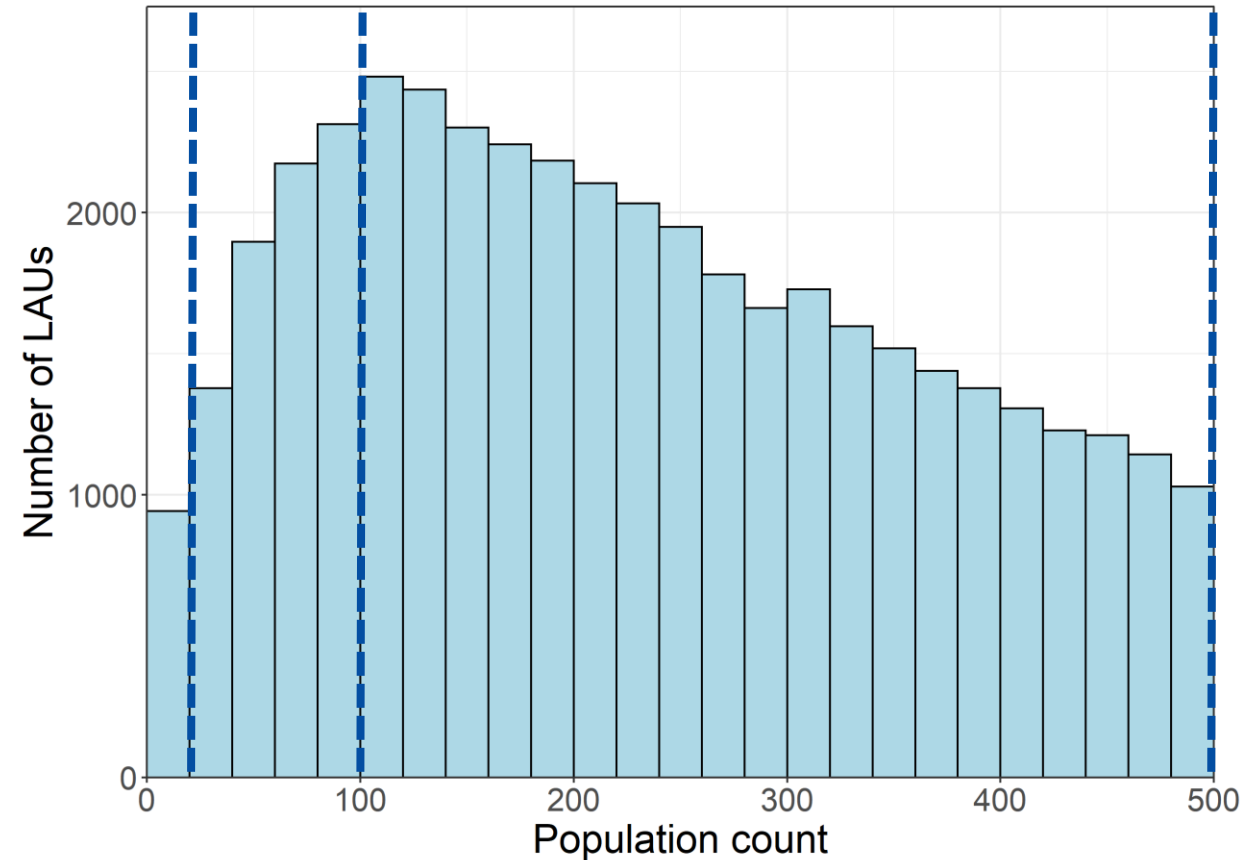


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☐ Yes

☒ No



# Utility: (noise) tail wagging the (statistic) dog

- mainly a problem of **strict  $\epsilon$ -DP** approaches

**Recall:** Noise magnitude bound parameter  $\epsilon$ , “cutting off” the tail, is **forbidden** in strict  $\epsilon$ -DP

- E.g. 2020 test setup of [U.S. Census Bureau \(2019\)](#) with moderate global  $\epsilon = 1$



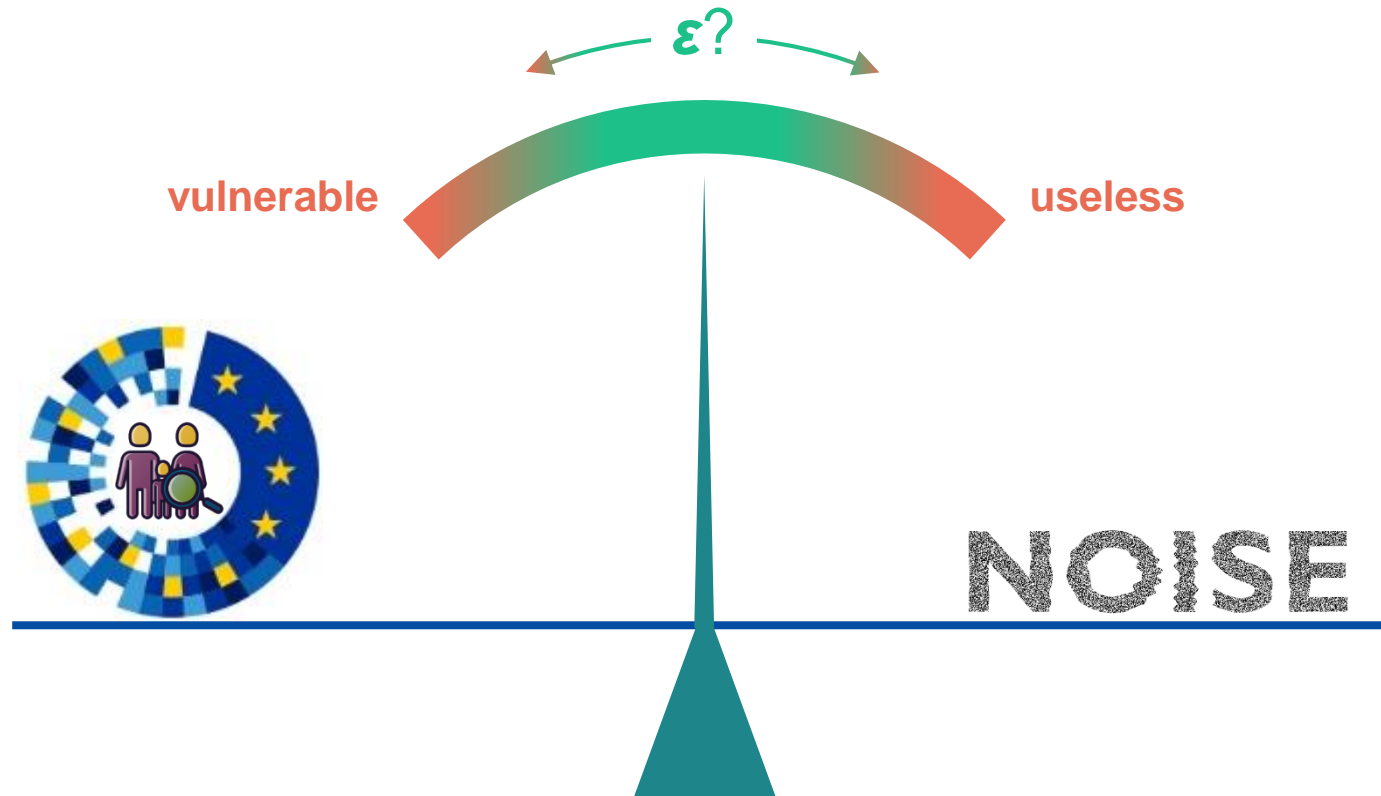
**Cidamón, La Rioja, Spain**  
ES230\_26048

	2011 census	strict $\epsilon$ -DP
Total	30	-17
Male	20	-1
Female	15	-9



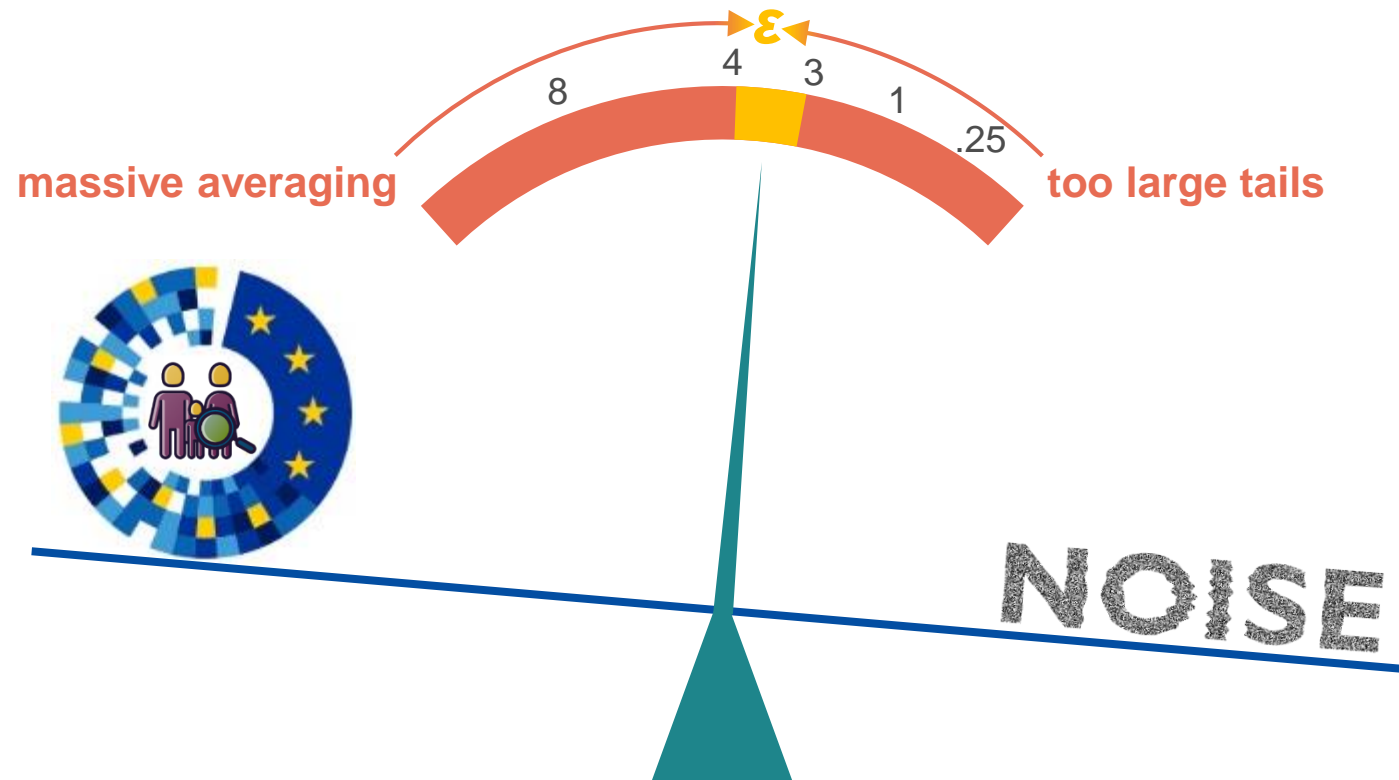
# Outro: the 2021 EU census picture

- risk + utility constraints on strict  $\epsilon$ -DP setup for whole 2021 EU census output



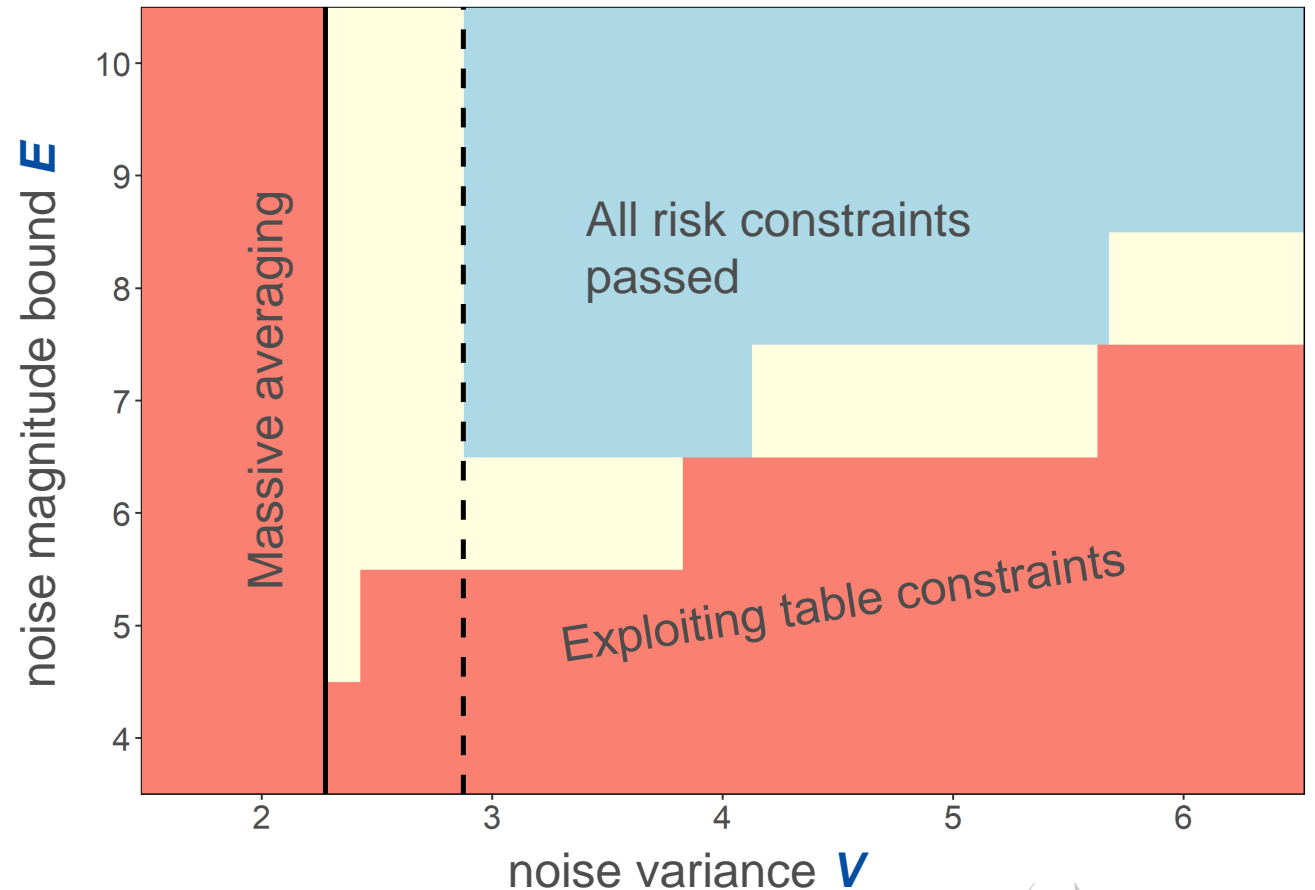
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# Outro: the 2021 EU census picture

- whole 2021 EU census output
- risk constraints on bottom-up parameter space  $V - E$
- utility controlled directly by  $V$  and  $E$  (utility-driven)
- e.g. cell key method recommended for 2021 EU census ([ESSnet, 2017, 2019](#))



# Thank you



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