Analytical solutions of the kinetic equation for rounded spirals in effectively isotropic systems

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A method for finding analytical solutions to the differential equation of N. Cabrera and M. Levin is proposed for the case of spiral formation on a screw dislocation, when there is no difference between the growth and evaporation of a crystal [1]. The differential equation describes the dependence of the angle between the position vector of the spiral and the tangent at that point on the dimensionless radius of the spiral. The equation parameter is the dimensionless angular velocity of the spiral rotation. The method for finding analytic solutions to the equation is based on finding asymptotic solutions for small dimensionless spiral radii (interval I) and exact solutions in the interval of radii greater than unity (interval II). Then it is used the method of matching the asymptotic and exact solutions and their derivatives for a radius whose dimensionless value is equal to unity.

An asymptotic solution of the original equation in interval I is obtained for the case of spiral nucleation at the origin, and its analytical solution in interval II is also found. The obtained solutions are used to describe the dependence of the angle between the radius vector of the spiral and the tangent at that point in the presence of a hollow nucleus. The obtained solution adequately describes the experimental data taken from the scientific publications. It is obtained a model solution of the original equation in interval II that satisfies the matching conditions with the solution in interval I for the spiral nucleating at the origin. In a particular case, a model solution of the original equation in interval II is the general solution of the Abel equation of the second kind. Comparison of the obtained dependence of the angle between the position vector of the spiral and the tangent at that point for the angular velocity $\omega_1 = 0.33$ with the results of calculations performed earlier by N. Cabrera and M. Levin shows their complete correspondence. It is obtained an expression for the dependence of the spiral angle on the radius in parametric form.

An asymptotic solution of the original equation is obtained in the interval of small spiral radii, i.e. for the case of the spiral nucleation outside the origin. Using the matching method, it is obtained the dependence of the angle between the position vector of the spiral and the tangent at that point on the spiral radius, where the parameter is the rotation angle. The example of synthetic diamond growth from the vapor phase shows that the spiral, whose nucleation point does not coincide with the origin, is described by the obtained parametric equations with a sufficient degree of accuracy.

The obtained analytical solutions make it possible to estimate the physical parameters of the crystalline medium by the type of spiral configuration and can be used in the analysis of rounded spirals in effectively isotropic systems (see, for example, [2] and references therein).

References

[1] Cabrera N, Levine MM. On the dislocation theory of evaporation of crystals. Phil. Mag. 1956;1(5):450-458.
[2] Miyamoto G, Kouchi A, Murata K, Nagashima K, Sazaki G. Growth kinetics of elementary spiral steps on ice prism faces grown in vapor and their temperature dependence. Cryst. Growth Des. 2022;22(11):6639–6646.